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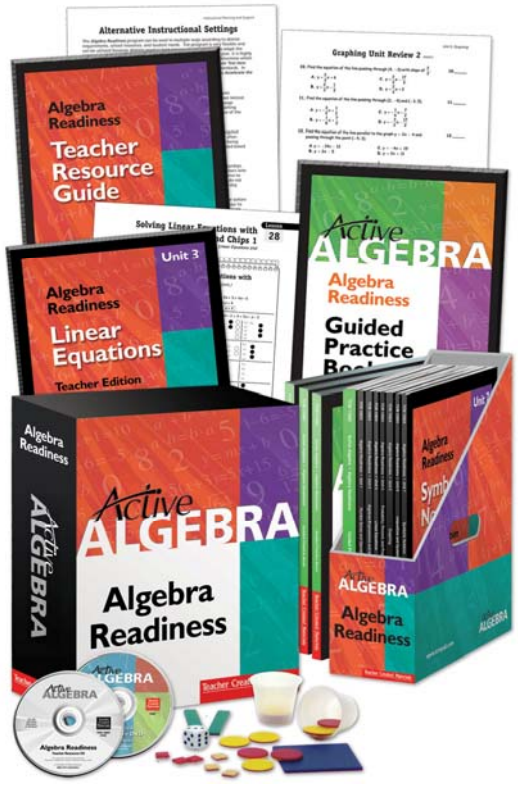
Active Algebra: Algebra Readiness



Introduction

Active Algebra: Algebra Readiness is a research-based intervention program designed for all students who are preparing to learn and master Algebra concepts. The program is intended to be a bridge from general mathematics to Algebra. Foundational mathematics skills and concepts are taught, practiced, and reviewed so that students can approach Algebra concepts with more confidence and certainty. "In order for students to develop the critical thinking skills necessary to pass Algebra, it is important for them to have a solid math foundation in the early grades (U.S. Department of Education, 2006). "Many students have difficulties making the transition from school arithmetic to school algebra—with its symbolism, equation solving, and emphasis on relationships among quantities. Recent calls of "algebra for all" have increased the number of students making the transition and therefore the number encountering obstacles" (Kilpatrick, et. al, 2003). One of the major reasons that so many students are failing Algebra nation-wide is that 90% of the objectives are brand new. If these students don't already have strong basic mathematics skills, they aren't prepared to move quickly through many of the necessary higher-level skills, most of which are abstract.

As today's society and economy change, the ways to successfully educate students must also change. It is critical that instructive methods, curriculum, and content be adapted to meet the current educational requirements expected for academically successful students. Educators must prepare students to participate in a global economy that relies on technology and products that are mathematically based. Classrooms today should provide students with the understanding, skills, and tools to succeed in a mathematically based world. Teachers must expect all students to learn mathematics well beyond what was previously expected. All students need to be more mathematically proficient than in the past, and many more students need to be encouraged to pursue careers in the fields of mathematics and science (Seeley, 2005). ***Active Algebra: Algebra Readiness*** lessons focus on helping students balance conceptual competence, procedural competence, problem solving, algebraic vocabulary, and mathematical reasoning in the areas of foundational mathematic basic skills that lead to targeted algebraic concepts.

Readiness versus Remediation

Customary treatment of mathematically struggling students is to place them in remedial or "make-up" courses *after* they have already failed to master the required content. "Current research on compensatory and remedial education programs shows that their goal of bringing academically deficient students back into the academic mainstream is not being accomplished. In fact, even though these programs are far more costly than regular programs and a whole lot of money is being spent on them, they remain unsuccessful for the long term and are only slightly effective for the marginal student. Most students involved in these programs actually become "lifers" without any hope of changing their circumstances" (Anderson and Pellicer, 1998). Mark Karadimos, who researched alternative options for Illinois students with high Algebra failure rates, stated: "When students fail classes, it breaks their potential for success. It interferes with the natural, seamless progression of mathematics courses because students who fail mathematics courses must often wait a semester before they can take a repeat course in the summer. Failing mathematics courses also undermines student confidence where it is already low due to poor ability and comprehension" (2004).

Rather than waiting to find out which students will require extra intervention and additional instruction in order to eventually pass the required Algebra classes, there is an increasing need for innovative programs that prepare students to comprehend algebraic concepts before they enter the Algebra I classroom. Programs that prepare students for college and higher education should focus on readiness rather than just remediation (Oesterreich, 2000). The recommendation logically follows that intervention and instructional support programs would serve students more effectively if they were closely integrated with the current math courses. These programs need to offer support and preparation for the concepts that students will be working on in the future. In this way, students would receive the additional support while mastering the necessary skills and concepts rather than after they had already failed to do so. ***Active Algebra: Algebra Readiness*** was designed with this in mind. The program can be used within current math courses in preparation for Algebra concept learning, concurrently with early Algebra courses, within math intervention programs for students who struggle with math, or as before/after school programs while the students are taking Algebra courses.

The Importance of Mastering Algebra Concepts

Algebra is often referred to as a gatekeeper to achieving the loftier societal goals. "Algebra is clearly the backbone of secondary mathematics. It furnishes concepts and symbolic conventions for representation of very important information in situations that affect each of us in obvious and subtle ways every day" (Christmas and Fey, 1990). It is necessary for students to master the concepts and skills in Algebra in order to take higher-level math classes and to pass college admissions exams. Many state departments of education realize the importance of fostering strong mathematics skills. For example, beginning with the class of 2006, California state law requires that all public school students pass the California High School Exit Examination (CAHSEE) in order to receive a high-school diploma (California Department of Education 2006). Texas is another state that lists the completion of Algebra I as a high school graduation requirement for all students entering ninth grade in 2001 or later (TEA, 2001). Furthermore, these types of math skills are required for students who wish to pursue careers in the areas of math and science.

Nevertheless, developing students' mathematical reasoning and algebraic thinking skills is not just the responsibility of secondary teachers. According to Margaret Spellings, the U.S. Secretary of Education in 2006, it is "more important than ever that our students receive solid math instruction in the early grades to prepare them to take and pass Algebra and other challenging courses in middle school and high school" (U.S. Department of Education, 2006). Researchers have assessed the mathematics standards taught at both the elementary and middle-school levels and analyzed the correlation between students' achievement on those standards and their success in Algebra I. This analysis shows that mastering certain foundational skills and concepts in the early grades contributes to students' success in Algebra I. "Mastery of algebra must be a K–12 undertaking. That is, the study and use of algebra must begin at the start of every student's formal schooling and must continue throughout each student's academic experience" (Strong and Cobb, 2000).

Algebra is no longer simply a one-time high school course that students struggle to pass. "What we need is a more continuous development of algebra from PK through grade 12" (Seeley, 2005). The Third Grade NCTM Focal Points (2006) state: "Understanding properties of multiplication and the relationship between multiplication and division is a part of algebra readiness that develops at grade 3. The creation and analysis of patterns and relationships involving multiplication and division should occur at this grade level." This is just one example of how primary grade teachers can prepare students for algebraic reasoning. These are also the types of foundational skills that ***Active Algebra: Algebra Readiness*** will reinforce. "The development of algebraic thinking is a process, not an event" (Seeley, September, 2004). It represents a lifelong engagement in mathematics.

Students who do not possess foundational algebraic thinking skills before entering Algebra I may have serious difficulties in the course. Algebra I can become a negative experience that destroys students' confidence in their mathematical abilities. "When students try to understand the abstract symbolism represented in algebra concepts without a foundation in how to think algebraically, they are likely to feel frustration and failure" (Seeley, September, 2004). To ensure lifelong learners and users of advanced mathematics, students need to participate in mathematical programs that develop a foundation in algebraic thinking and lead them to discover the abstract symbolism of algebra. The Mathematics Framework for California Public Schools states, "It is imperative for students, whether in grade eight, grade nine, or even a later grade, to master prealgebraic skills and concepts before they enroll in a course that meets or exceeds the rigor of the content standards for Algebra I" (2006). The U.S. Department of Education also believes this is true. "It is crucial that middle school students who are significantly below grade level in math receive appropriate and effective interventions so that they will be prepared to take challenging math courses in high school" (U.S. Department of Education, 2006).

Foundational Skills and Targeted Standards

Specific curriculum needs to be used with students to address these prealgebraic skills and concepts. Such a program should include a small subset of standards from the elementary- and middle-school math curriculum that truly focuses on preparing students for success versus attempting to remediate and reteach all the math standards for kindergarten through seventh grade. Students' acquaintance with Algebra begins in elementary school with an attentiveness and understanding of patterns, relations, equality, and change over time (Florian and Dean, 2001). It is essential for younger students to build a foundation for more complex materials in later grades.

Active Algebra: Algebra Readiness meets these needs in many ways. The program serves as a bridge between general mathematics and Algebra I by providing comprehensive instruction of the prerequisite standards necessary for students who are not prepared for Algebra I in grade eight. The program includes a unit dedicated to developing conceptual understanding and procedural proficiency for the foundational skills and concepts that were taught in grades three through five. Further, these foundational skills are embedded throughout the rest of the lessons. They often serve as review prior to introducing related prealgebra standards. In further units, the lessons work on the targeted standards for Algebra readiness. In these lessons, there is often review of the foundational skills prior to the introduction of the related targeted standards.

Important mathematical ideas need to continually be revisited so students can develop a healthy understanding of mathematics (Coxford, et al. 1998). The program consists of 95 lessons, as well as 7 further lessons that explicitly focus on problem solving strategies. In addition, bonus activity sheets and multiple forms of assessment are included on the Teacher Resource CD for reteaching and intervention support of these foundational skills, as well as the targeted standards.

The program also correlates with the National Council of Teachers of Mathematics (NCTM) Standards. It merges the most necessary foundational mathematics concepts and skills from grades three through five with the preparation for learning the targeted standards for grades six through nine, along with some Algebra I standards. The National Council of Teachers of Mathematics (NCTM), in the 2000 document entitled *Principles and Standards for School Mathematics*, claims that Algebraic symbols and procedural knowledge of Algebra are an immense mathematical accomplishment and that comprehension of these is essential in mathematical work. While some think that these concepts are most appropriately taught in middle school or high school, NCTM maintains that "even young children can be encouraged to use Algebraic reasoning as they study numbers and operations and as they investigate patterns and relations among sets of numbers" (Principles and Standards for School Mathematics Executive Summary, 2000).

These skills are addressed repeatedly in the lessons throughout the ***Active Algebra: Algebra Readiness*** program. Robert Marzano (2003) states that students should have multiple experiences with topics allowing them to integrate the topics into their knowledge bases. Therefore, the targeted standards are not simply taught, practiced, and assessed in one lesson. The scope and sequence of the program is designed in such a way that the standards are taught, reviewed, and practiced in multiple lessons before being assessed. To ensure depth of coverage, additional review activity sheets are also included on the Teacher Resource CD.

Three Phases of Instruction

The scope and sequence of ***Active Algebra: Algebra Readiness*** is not only innovative in the way that it continuously reviews both foundational skills for grades three through five and targeted standards for grades six through nine, but it also offers a blend of instruction that moves students' understanding of a given concept from the concrete through procedural proficiency to real-life application. This process allows time for students to see multiple representations of given concepts and develop mathematical reasoning. These three phases of instruction are interdependent. Without conceptual understanding, students do not understand when to use certain procedures or why they are using them. Equally as important is procedural proficiency. Whereas conceptual understanding is knowing what do, procedural proficiency is knowing how to do it (California DOE, 2006). Siegler and Stern (1998) state that "conceptual understanding provides important constraints to the types of procedures children use to solve mathematics problems; at the same time practicing procedures provides an opportunity to make inductions about the underlying concepts." Last but not least, real-life application and problem solving play an important role in achieving an understanding of the big picture of mathematics. Problem solving is the marriage of conceptual and procedural competencies. It is knowing where and when to use the procedures (California DOE, 2006). Research shows that teachers need to provide students with opportunities to interact in problem-rich situations where they find their

own solution methods and are given opportunities to share and compare solution methods and answers (Grouws and Cebulla, 2000). ***Active Algebra: Algebra Readiness*** reflects this balance of conceptual competence, procedural competence, and mathematical reasoning. The lessons embrace procedure, practice, and note-taking along with problem solving, game, and manipulative practice opportunities.

Conceptual Understanding

Active Algebra: Algebra Readiness uses manipulatives, visual aids, and other hands-on activities in order to develop conceptual understanding. Manipulatives serve as a great tool for engaging students as well as addressing the needs of kinesthetic learners, visual learners, and English language learners (ELLs). Using manipulatives regularly provides hands-on experiences and helps students construct useful meanings for the mathematical concepts they are learning (Grouws and Cebulla, 2000). They lead to larger mathematical learning goals (Kilpatrick, et al., 2003). The use of manipulatives has become common in the primary grades and has proven to be an effective tool for illustrating math concepts. Manipulatives, such as algebra tiles, have extended this physical representation into Algebra I and provide a basis for developing algebraic concepts (Sharp, 1995). Using manipulatives provides universal access to the algebra readiness curriculum. When students use manipulatives, they perform better academically and have more positive attitudes toward algebra (Leinenbach and Raymond, 1996).

Graphing calculators are integrated into the ***Active Algebra: Algebra Readiness*** program. They, like manipulatives, promote conceptual understanding. The graphing calculator is a device that provides universal access to the curriculum. With proper use, graphing calculators can meet the needs of all students by promoting higher levels of thinking, increasing student performance in math, and allowing access to mathematical exploration, experimentation, and enhancement of mathematical concepts (Waits and Pomerantz, 1997). The use of graphing calculators allows students access to concepts and mathematical problems that may not be understandable or solvable with paper and pencil alone. Graphing calculators also help to build procedural competence. They facilitate improvement in procedural fluency and the ability to compute, calculate, and use rules and formulas accurately with speed and confidence (Florian and Dean, 2001).

Moving to Procedures

Following the development of conceptual competence through manipulatives and graphic calculator use, ***Active Algebra: Algebra Readiness*** develops procedural competence. Students use the understanding they gain from their experiences with the manipulatives to learn the procedures and algorithms that accompany the concepts. These procedures and algorithms are modeled through a four-stage process: I model, you watch, I model, you help; you model, I help; you model, I watch. Teachers are guided through these stages in the teacher directed notes available for each lesson plan. Through this approach, students practice the numerous representations of concepts and experiences in a way that is not possible by paper and pencil alone. As a result of these methods, teachers are able to engage students more effectively by addressing different learning styles and developing understanding that leads to higher-level thinking. As students move through each phase of learning, they are exposed to a concept or skill numerous times. The notes for key concepts are also provided in PowerPoint slide shows that

accompany many of the lessons. The slide shows support ELLs and visual learners, and they are an exceptional tool for reteaching lessons. Teacher-directed note taking is a routine that gives students a consistent structure and format for instruction and practice. Mathematics notes help solidify students' connections between the procedures and concepts (Seeley, November, 2004).

This program also uses a ***Student Guided Practice Book*** to guide students through practice problems. The use of the book allows students and teachers to easily differentiate between practice and assessment. Students need ample opportunity to practice in order to be able to execute procedures automatically without conscious thought (Kilpatrick, et al., 2003). When students are working on guided-practice problems without the pressure of being assessed, they are more likely to take risks with the procedures and problem-solving strategies they use. In guided practice, "students' strategies yield to teachers' questions, but the solution is more than the answer, just as the problem is more than the question. Generating a strategy and arguing for its legitimacy indicates what the student knows about mathematics" (Lampert, 1990).

Application of Problem Solving

The third phase of instruction is the real-life application of the concepts and procedures. Application of the learning during the problem-solving stage provides the foundation for students' mathematical reasoning. Research has shown that real-life applied activities and problem-solving activities establish a contextual setting for many lessons, providing motivation and encouraging curiosity (Hiebert and Carpenter, 1992). When students are given sufficient practice, they can approach being able to use the newly learned skill in new situations with accuracy so that that skill will be retained (Sousa, 2006). In ***Active Algebra: Algebra Readiness***, these activities are integrated into the lessons in the notes, transparencies, and ***StudentGuided Practice Book***. The lessons provide opportunities to model and practice problems in a variety of instructional groupings. Furthermore, at the end of each unit is a focused problem-solving lesson that introduces a new problem-solving strategy and gives students an opportunity to apply the strategy in multiple real-life application activities. Teachers are given an opportunity to explicitly teach problem solving strategies that can later be applied to other problems. It is important that problem-solving experiences are meaningful and interesting so students are more likely to experience success and develop confidence in their mathematical reasoning skills. In this program, these strong problem-solving activities are included directly with the units.

Student Engagement

Besides the key instructional strategies mentioned above, other activities are integrated into the lessons to engage students in the curriculum. These activities include games, art, and technology, as well as cooperative learning. "Student engagement is perhaps our most important tool in our battle for equity" (Seeley, November, 2004). Engaging students helps break the detrimental cycle of "business as usual" in the mathematics classroom. When students are engaged in the learning process, they are actively constructing knowledge. Students are more likely to have ownership of the content. Additionally, by using such methods, teachers better address different learning styles and develop the understanding that leads to higher-level thinking. When students are actively involved in writing, modeling, exploring, and discussing mathematics, versus watching the teacher do these things, students are more likely to be successful (Seeley, November, 2004). ***Active Algebra, Algebra Readiness*** makes use of game activities that are available on the

Teacher Resource CD. Playing games with the goal of reinforcing skills, rehearsing information, and building retention of Algebra concepts is one way to accomplish this goal. Time on-task is high. "Besides being fun, it has the added value of making students rehearse and understand the concepts in order to create the quiz questions and answers" (Sousa, 2006).

Differentiation

Today's students have different learning styles, come from different cultures, experience a variety of emotions, and have varied interests. As a result, they differ in academic readiness. To better meet these diverse needs, teachers must differentiate their teaching. Carol Ann Tomlinson (2000), the leading researcher on differentiation, says, "Differentiation is simply a teacher attending to the learning needs of a particular student or small group of students, rather than teaching a class as though all individuals in it were basically alike." There is not just one plan for how to successfully differentiate. Differentiation has multiple faces, depending on the particular students and teachers involved, the outcomes of these learners, and the structure of the classroom environment (Pettig, 2000).

Differentiation encompasses what is taught, how it is taught, and the products students create to show what they have learned. Strategies for differentiating content, process, and products for English language learners, struggling students, and advanced learners are provided in the Teacher Resource Guide. To vary the degree of support for below-level students, additional strategies are provided for addressing the needs of benchmark, strategic, and intensive students. Manipulatives, teacher-directed notes, multiple problem-solving strategies, and other tools for engaging students are strategies that make the content accessible to all students. To further support differentiation of the lessons, strategies for involving parents, grouping students, engaging students, and introducing vocabulary are also provided in the Teacher Resource Guide.

Vocabulary Development

In math, vocabulary is highly specialized. It is not enough to give the students a list of words and have them look up the definitions in dictionaries or glossaries. Students who are struggling with learning a language, in particular, are not going to find the process easier by simply being given more words to sort through (Echevarria, Vogt, and Short, 2004). What English Language Learners need are context-embedded lesson activities that acquaint them with the necessary words for comprehension of the content and allow them to practice the use of the words in activities that span listening, speaking, reading and writing actions. Math content vocabulary words are often not encountered in everyday life. Also, these are not typically the words that ELLs will learn during their structured English Language Development class period. These words have particular technical uses and even a syntax form that is specific to the math content area. "Instead of being 'language neutral,' mathematic vocabulary poses numerous problems for English learners (Diaz-Rico and Weed, 2002). ELLs not only have the difficult task of gaining proficiency with the mathematical skills, but they are also simultaneously acquiring social language, academic language, and content-area language. "The language of mathematics is very precise compared with the English used in common discourse, and this difference separates mathematics from most other curricular areas" (California DOE, 2006).

Therefore, it is up to the content teacher who teaches math to make certain that English language

learners learn the necessary vocabulary in order to achieve comprehension of mathematical concepts and curriculum. This is especially true for Algebra concepts because Algebra is a graduation requirement in most states. Knowing the necessary vocabulary allows access to the core curriculum. English Language Learners need to demonstrate mastery of the concepts and this will only be possible if they first achieve understanding of the vocabulary words that explain, describe, and facilitate each of the foundational and targeted algebraic concepts. To further address the needs of the ELLs, the Strategies for Universal Access section of the Teacher Resource Guide includes specific information for this group of students. The section, Vocabulary Development for English Language Learners includes levels of language proficiency and suggestions for addressing students' needs at each level. There are ten short activities for building vocabulary with suggestions for implementation. These can be taught in progression or over time and used repeatedly throughout the lessons in the program. Rather than making teachers guess at the most important words to use for vocabulary development, the Teacher Resource Guide provides lists of the vital content vocabulary words for each specific lesson. Graphic organizers for the activities are also provided to facilitate trouble-free teacher use in the classroom. In addition, this section includes some general strategies for developing the language of mathematics. The language of mathematics is a "foreign" language for all students, and all students will benefit from activities that help them acquire that language.

Assessment

Finally, embedded into the scope and sequence of the program are assessments that align with the skills and concepts taught throughout the program. The ***Active Algebra: Algebra Readiness*** Diagnostic Test is given as an entry-level assessment at the beginning of the program to determine students' levels. It includes an item-by-item analysis that correlates the questions to the lessons. With this item analysis, teachers can evaluate their students' work and determine which lessons need the most focus. This assessment is also given as a progress monitoring assessment and as a summative evaluation to measure students' growth. Additional progress monitoring assessments include the quizzes and practice activities. Other summative evaluations include the unit tests. Two forms of each assessment are provided for retesting or for makeup assessments. The data from all the assessments can be analyzed and utilized to show students' growth. The Data-Driven Instruction section of this book guides teachers in using the assessments to effectively meet the goals of this program.

Conclusion

Hassler Whitney (1985) states, "Students should grow in their natural powers of seeing mathematical elements in a situation, reasoning with these elements to come to relevant conclusions, and carrying out the process with confidence and responsibility." ***Active Algebra: Algebra Readiness*** supports this goal. This program develops students' conceptual competence, procedural competence, and mathematical reasoning by using a balanced approach, best practices of mathematics instruction, and effective strategies for universal access. With the completion of the algebra readiness curriculum and the mastering of the algebra readiness foundational skills and targeted standards, students will have the keys to unlock the gates of Algebra I and higher-level math.

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