

Sample Pages from *Exploring Math: An Intervention & Reinforcement Resource*

- Teacher Resource Guide Introduction
- Unit 1: Numbers and Numeration
- Problem Solving Strategy Card: Looking for a Pattern
- Real Life Problem Solving Card: Going to the Movies

Introduction

Exploring Math: An Intervention and Reinforcement Resource is a mathematics program that offers intervention and reinforcement opportunities in key areas of the math curriculum. This program can be used for summer school, after school and tutoring programs, or to support a year-long mathematics program.

Each kit includes:

- ❑ Teacher Resource Guide
- ❑ 7 units of lessons, with each unit focused on a specific set of content and process skills
- ❑ 7 Problem-Solving Strategy Cards, to be used with problem-solving lessons in the units
- ❑ 14 Real-Life Problem-Solving Cards, to be used with problem-solving lessons in the units
- ❑ 14 Skill Application Games to reinforce skills introduced in unit lessons
- ❑ CD-ROM with downloadable practice pages, tests, problem-solving and strategy cards, reproducibles, and PowerPoint® Lessons
- ❑ 42 transparencies of problem-solving and strategy cards, for use with problem-solving lessons

The Teacher Resource Guide

The guide provides the teacher with a variety of tools and information to use with the materials in this kit. It is divided into these sections:

- ✓ Introduction
- ✓ Management (includes standards correlations and a listing of objectives for each unit)
- ✓ Placement Test
- ✓ Problem-Solving Strategies
- ✓ Skill Application Games
- ✓ Glossary (a teacher/student guide to terms commonly used in the units)

The units in this kit contain specific, sequential lessons by topic, and are divided into units that target the following content goals and processes:

Numbers, Operations, and Computation
Patterns and Algebraic Thinking
Geometry
Measurement
Data Analysis
Problem-Solving
Mathematical Reasoning and Proof

Management

Exploring Math: Intervention and Reinforcement Resource allows the teacher to focus on math topics in which students need additional reinforcement. It is especially important in intervention and other support programs to determine areas of greatest need as soon as possible.

⇒ Getting Started: The Placement Test

The first step in using the materials in this kit is to administer the Placement Test. The results will help you evaluate your students' skill levels in each of the units in the kit. We have provided a placement test in this guide (pages 47–67) that should be administered prior to formal instruction. The placement test consists of 15 items for each unit in the kit. Test items are representative of the unit's contents, and have been named Unit 1, Unit 2, Unit 3, and so on, to correlate with the unit guides.

Reproduce a copy of the test for each student. Students can write on the copy (circling the appropriate answer choices), or, the test copy can be used for reading only, and students can use the bubble-in test

answer sheets found on pages 68 and 69. In either case, students should show their work on the test, or on separate paper. An Answer Key is provided on page 70.

You may wish to administer the test in two or three sessions. This allows time for scoring between test-taking sessions. A Placement Test Class Record is available on pages 71 and 72. Record student names and test results on this form for an overview of class test results. After a test is scored, write in the number of correct responses for each student in the appropriate columns. For example, if Johanna scored 11 out of 15 correct on the Multiplication and Division test items, write in “11” in the top section of the scoring box for that unit.

When all sections of the test have been administered, and the students' scores are recorded, review the test score record

form. First, find the mean for each unit by averaging the columns and writing the average at the bottom of each column. The results will provide you with an overview of the students' overall performance and help you prioritize the unit(s) that need the most reinforcement. If mean scores are similar, and the overall results are fairly equal, plan to start with the math topic that is generally most essential to your math curriculum needs.

Evaluation of the placement test results will help you establish the unit(s) on which to focus attention. If, for example, students score lowest on a set of multiplication and division items in the placement test, you should consider the Multiplication and Division unit a priority. See pages 25–27 for pacing plans.

Placement Test Unit 1

Name: _____

Date: _____

The questions on the following pages will help you and your teacher find out which math skills you are already good at and which ones you need to work on.

To help you do your best, remember to do the following:

- Read the questions and answer choices carefully.
- Circle the letter that shows the answer you have chosen. (If your teacher provides you with an answer sheet, fill in the circle next to the correct answer on the answer sheet.)
- Show your work in the question box. Use another sheet of paper if you need more room.
- Try completing all questions.

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Management

⇒ Getting Started: The Placement Test (cont.)

It is recommended that you keep a file for each student, in which to store placement tests, pre-tests and post-tests, and other documentation of progress the student is making as he or she moves through the unit(s) in the kit.

⇒ Using the Lessons

After the placement test scores have been recorded and reviewed, select the unit(s) you feel need the most reinforcement. Look over the information in the written lessons or the PowerPoint® lesson presentations.

The Tests

Before you start a lesson, give the students the pre-test for that lesson. The pre-test can be administered at the end of a previous lesson, at the beginning of the day, or at some other point in your schedule that will allow you time to score it. Because the test is short, scoring time is minimal. (Information on the tests, preparing individual student files, and record-keeping forms are provided on pages 8–11 of each unit.)

To prepare the tests, reproduce copies of each for the students. Cut the copies in half along the dashed line, separating the pre-tests from the post-tests. Store the tests until needed. (Copy all the unit tests at once in this way, or only prepare a few units at a time.)

Use pre-test results to evaluate students' needs as they relate to the content of the lesson, and to determine your student groups during the Differentiated-Group Skills Practice portion of the lesson. Since there will inevitably be variations in ability within each group, some flexibility is advised in assigning group work. However, the pre-tests can serve as a tool for determining initial grouping for the lesson.

Overall assessment of each student's progress can be checked with the post-test, administered at the end of the lesson. Record scores on the record-keeping form provided in each of the units.

Unit Organization and Resources

Student Test Scores _____ Name _____

Unit: _____

Lesson	Learning Objectives	Unit Scores			
		Pre	Post	M	A
1.1	Read and write numbers with ones, tens and hundreds. Identify the place value of each digit.	3	3		
1.2	Read and write numbers to the millions place. Identify the place value of each digit of a number to the millions place.	3	3		
1.3	Compare whole numbers and numbers less than one million. Read and write numbers with ones, tens and hundreds.	3	3		
1.4	Recognize equivalent fractions. Order a set of positive and negative numbers.	3	3		
1.5	Use a number line to add and subtract signed numbers. Order a set of negative numbers on a number line. Round, estimate and compare whole numbers to the nearest hundred.	3	3		
1.6	Round a number to the nearest 10, 100, and 1,000. Estimate to find an approximate product.	3	3		
1.7	Use rounding to estimate an unknown. Estimate to find an approximate quotient. Recognize, estimate, and compare real number plans.	3	3		
1.8	Round a number to the nearest 10, 100, or 1,000. Round a number to the nearest 10, 100, or 1,000.	3	3		
1.9	Divide to find an approximate quotient. Round a number to the nearest 10, 100, or 1,000.	3	3		
1.10	Use a number line to estimate. Identify prime numbers.	3	3		

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Introduction

The Units

Each unit is organized as follows:

Table of Contents and Introductory Pages

Lessons

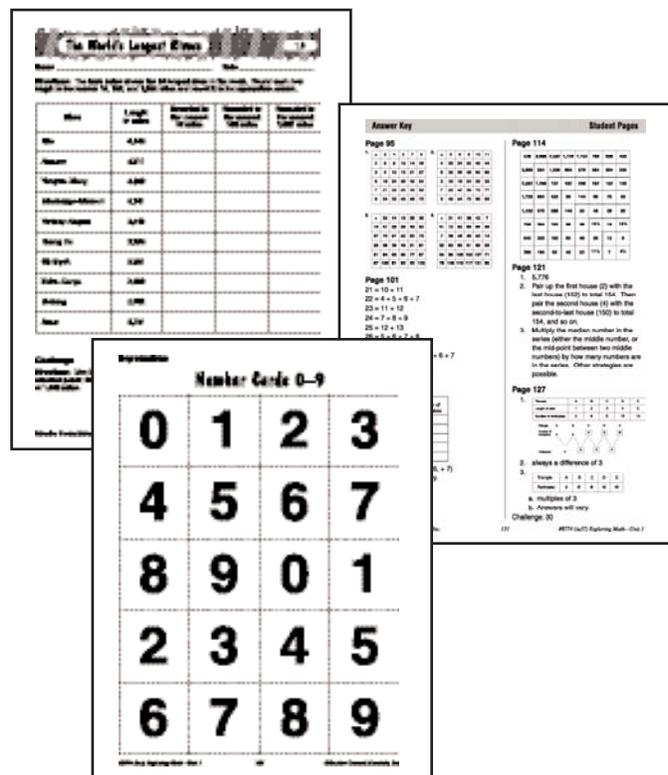
- Teacher Lesson
- Student Page(s)
- Pre-Test/Post-Test Page

Answer Keys

- Keys for Student Pages
- Keys for Pre-Tests/Post-Tests

Reproducible Pages

- commonly-used reference pages



Every unit in the Exploring Math kit contains the lessons, student activity pages, pre-tests and post-tests, answer keys, and reproducibles for the unit. Since the units are self-contained and topic-specific, they are also ideal for use as replacement units in the classroom.

Note: With each lesson, the teacher has the option of choosing from two forms of presentations. The first method is to use the lessons as written in the unit. For those who wish to use an alternative method, we have provided PowerPoint® slide shows that meet the same lesson objectives, and include warm-up activities and the main lesson as well. A thumbnail copy of the PowerPoint® slides is included in the lessons, and each PowerPoint® lesson presentation can be found on the CD-ROM.

The administration of pre-tests and post-tests provides a method of assessing student achievement. Pre-tests should be given prior to each lesson, to assist the teacher in determining classroom needs. Forms for recording test scores for each student can be found at the beginning section of each unit.

Answer keys for both the student pages and the pre-tests and post-tests are located within that unit. The student page answer keys are presented first, followed by the test answer keys.

The last section of each unit contains reproducible pages that may be commonly used throughout the unit. Material on some of the pages are used as manipulatives. Other pages provide reference information for the students.

The following pages provide additional details of the lesson content and organization.

Introduction

The Units: Lesson Organization

The lesson is identified by number. The first number is the unit, the second number represents the lesson.

Each lesson identifies the topic area or skill to be covered in the lesson.

A visual representation of the lesson tests, student pages, and reproducibles is provided.

Focused learning objectives are identified. A standards correlations chart, located in the Teacher Resource Guide, matches lesson objectives to national standards.

The clock indicates approximate times for the activity. Times are provided for the warm up, the skills lesson, and the group work activities.

Lesson 1.3 **Place Value and Ordering Numbers** **Unit 1**

Learning Objectives

- ✓ Use place value to sort numbers into ascending order.
- ✓ Read and write numbers with seven digits or less.

Resources

- Number Cards 0–9 (page 136)
- calculators
- “Raffle Tickets” (page 31)
- Pre-Test/Post-Test (page 32)

Number Cards 0–9

0	1	2	3
4	5	6	7
8	9	0	1

Warm-Up Activity *Mental Math and Number Sense Skills*

1. With the class, practice counting forward and backward by thousands from three-, four-, and five-digit numbers.
2. Provide an example number, such as 9,310, and ask students to count forward together, by thousands, to 19,310.
3. Provide an example number, such as 56,224, and ask students to count backward together, by thousands, to 42,224.
4. Continue assigning starting numbers and ask students to count forward or backward in thousands. In each case, give the class an appropriate ending number.

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Lessons identify specific resources, including any related practice pages from the CD-ROM, reproducible pages, game-board activities, or suggested classroom manipulatives.

Note: For PowerPoint® lesson options, arrange for the necessary equipment in advance.

The Warm-Up Activity reinforces mental math and number skills and actively engages students prior to the lesson. Activities help develop students’ abilities to think mathematically. Content may or may not link directly to the main body of the lesson.

Note: As with the main lesson and group work, the alternative PowerPoint® warm-up lesson can replace this written lesson.

Introduction

The Units: Lesson Organization

This is the whole-class or whole-group section of the lesson. The interactive lesson focuses on the skills listed in the learning objectives. The text includes questions to ask the students and examples that reinforce the skills being taught. The Whole-Class Skills Lesson is a suggested plan for teaching the objectives. You can adapt the contents to meet the needs of your students.

Unit 1 **Place Value and Ordering Numbers** **Lesson 1.3**

Whole-Class Skills Lesson

If the Pre-Test (page 32) has not yet been given, administer it at this time. (See pages 8–11 for testing information.)

Prepare to group students for the Differentiated-Group Skills Practice which follows the Whole-Class Lesson.

Use the directions below, or the PowerPoint® presentation (see PowerPoint® Lesson Option preview, page 30), to teach this lesson.

1. Tell students, "Today we will be arranging numbers in sequential order by looking at the place value of the digits."
2. Write three, three-digit numbers on the board, and demonstrate how to write them in ascending order looking at the number in the hundreds place first, then the tens, and finally the ones. (Discuss the term *ascending*, using examples from real life, such as ascending a mountain or staircase.)
3. Show students how to use the method described in step 2 to order four- and five-digit numbers.
4. Give students some examples of ordering numbers where one or more of the digits are the same and require them to look further at the place value.
Examples: 7②1 7④2 7①5
ascending order: 715; 721; 742
4,⑥5 2 4,⑦9 1 4,⑥3 8
ascending order: 4,638; 4,652; 4,791
5. Continue practicing this method with additional examples like the ones above.

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Reminders about pre-test administration and preparation for Differentiated-Group activities are provided.

PowerPoint® Lesson option information is listed here.

The text is numbered, providing easy-to-follow steps as the lesson is presented.

Introduction

The Units: Lesson Organization

Use the differentiated-group practice suggestions to reinforce the skills taught in the lesson. Groups can be determined by pre-test results and/or general understanding of the concepts and skills introduced in the lesson. The Differentiated-Group Skills Practice portion of the lesson provides an opportunity for more focused learning and addresses the various ability levels within the class.

The symbols indicate levels of difficulty (On Level, Below Level, Above Level).

Suggestions for the Lesson Review are included. They provide an opportunity for group discussion and reflection.

A Post-Test reminder is included. Use test results to assess students' understanding of the skills and concepts introduced in the lesson.

Check here for any related practices pages from the CD-ROM and/or Skill Application Game cards that reinforce some of the skills taught in the lesson.

Some group activities are facilitated by the teacher, while other group practice is assigned as small group or independent work. The method of presentation is noted within the text.

These optional activities may include a homework assignment or activity that challenges students to extend their thinking.

Lesson 1.3 **Place Value and Ordering Numbers** **Unit 1**

35 **Differentiated-Group Skills Practice**

Above Level – Teacher Directed

- Have students explore place value up to millions. Read aloud a seven-digit number, and have them write the number.
- Ask students to identify what each digit represents and also to solve number sentences such as the following:
 $1,264,521 = 1,000,000 + 200,000 + \underline{\quad} + 4,000 + 500 + 20 + 1$
- Instruct students to use number cards 0–9 to make the largest and smallest seven-digit numbers possible, and then write the numbers in both digits and words. Once finished, ask students to complete the activity sheet independently.

Below Level – Teacher/Student Directed

- Have students choose three number cards 0–9. Tell them to arrange the three cards to make as many three-digit numbers as they can and write the numbers on a piece of paper.
- Have students write the numbers in ascending order.
- Let students choose three different number cards and repeat the process. Have students complete the activity sheet in pairs.

On Level– Student Directed

- Students should complete the activity sheet independently.

Lesson Review

Reinforce how to sort numbers into ascending order by writing several numbers on the board. Tell students to write them in order on a piece of paper. Review the answers together.

Extension

Extend the lesson by instructing students to use all of the number cards 0–9 (page 136) to make two five-digit numbers that have the smallest difference. (90, 123 – 87, 654)

Additional Extension Resources:

- TRG Practices: 2, 13, 17, and 25

Administer the Post-Test (page 32) at this time. (See pages 8–11 for testing information.)

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Introduction

The Units: Lesson Organization

With the exception of the two strategy-based problem-solving lessons, the teacher has the option of choosing from two forms of presentation. The first method is to use the lesson as written in the unit. For those who wish to use an alternative method, we have provided PowerPoint® slide shows that meet the same lesson objectives. If you decide to use the PowerPoint® lesson option, keep in mind that you will need the same resources and general preparation as in the written lesson.

PowerPoint Lesson Option Lesson 1.3

<p style="text-align: center;">Today's Lesson</p> <p style="text-align: center;">Place Value and Ordering Numbers</p>	<p style="text-align: center;">Warm-Up Activity</p> <p style="text-align: center;">We will warm up today by counting forward and backward in thousands.</p>	<p style="text-align: center;">Re-examine the ones in thousands together. Place the strategy numbers.</p> <p style="text-align: center;">5,625 9,214 63,425 96,342</p>	<p style="text-align: center;">Whole-Class Skills Lesson</p> <p style="text-align: center;">Today we will be arranging numbers in sequential order by looking at place value.</p>
<p style="text-align: center;">438 421 426</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Look first at the hundreds digit.</p> <p style="text-align: center;">Each number has a 4 in the hundreds.</p>	<p style="text-align: center;">438 421 426</p> <p style="text-align: center;">Now look at the tens digit.</p> <p style="text-align: center;">We can start to order the numbers.</p> <p style="text-align: center;">426 421 438</p>	<p style="text-align: center;">Finally, use the ones column to order your numbers.</p> <p style="text-align: center;">426 421 438</p> <p style="text-align: center;">421 426 438</p>	<p style="text-align: center;">Top Tip</p> <p style="text-align: center;">Remember to order three-digit numbers by looking at the hundreds column, then the tens column, then the ones column.</p>
<p style="text-align: center;">256 272 252</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">252 256 272</p>	<p style="text-align: center;">356 472 372</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">356 372 472</p>	<p style="text-align: center;">5,791 5,652 5,638</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">5,638 5,652 5,791</p>	<p style="text-align: center;">3,791 3,692 3,638</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">3,638 3,692 3,791</p>
<p style="text-align: center;">9,791 9,971 9,719</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">9,719 9,791 9,971</p>	<p style="text-align: center;">78,719 74,971 79,971</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">74,971 78,719 79,971</p>	<p style="text-align: center;">62,418 62,148 62,841</p> <p style="text-align: center;">Order these numbers.</p> <p style="text-align: center;">Smallest Largest</p> <p style="text-align: center;">62,148 62,418 62,841</p>	<p style="text-align: center;">Group Practice</p> <ul style="list-style-type: none"> Identify each digit in a seven-digit number. Practice using a place value chart. Practice ordering four and five-digit numbers.

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The slides present only the Warm-Up Activity and Whole-Class Skills Lesson components.

The slide presentation meets the common objective(s) of the lesson, but the content and examples are not exact duplicates of the written lesson. It is an alternative plan for teaching the warm ups and main lessons.

Suggestion: If the written lesson has been given, and students need further review, consider using the PowerPoint® Lesson Option as additional reinforcement.

Once you have completed the Warm-Up Activity and the Whole-Class skill slides, you are directed to the group practice activities. The last slide indicates what each group will focus on as they practice and reinforce the skills learned in the main lesson. This is only an introduction to the activities. For group details, see the information provided in the Differentiated-Group Skills Practice section of the written lesson.

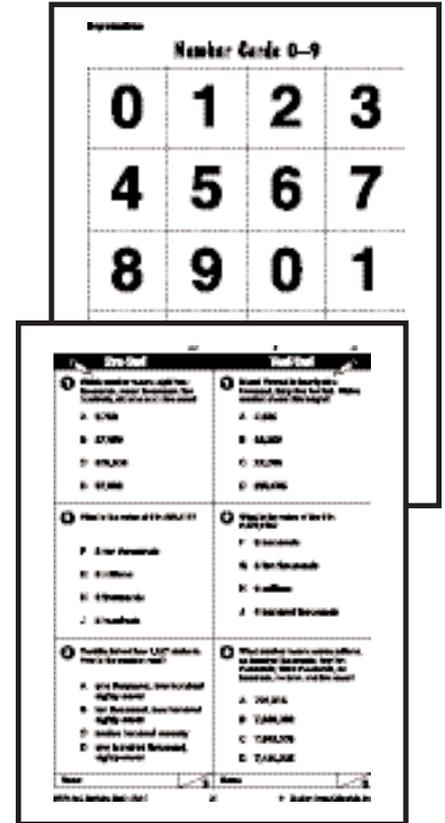
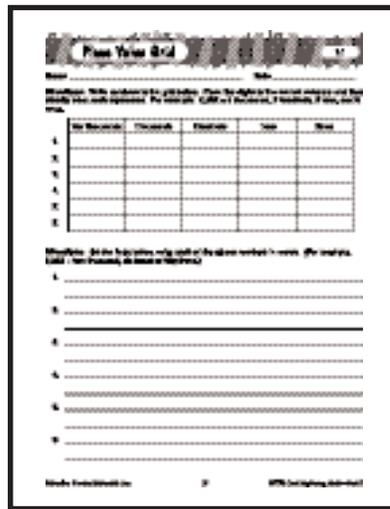
Following the group practice activities, continue as you would with the written lesson by completing the Lesson Review, administering the Post-Test, and introducing the optional Extension Activities.

Learning Objectives

- ✓ Read and write numbers with seven digits or less.
- ✓ Identify the place value of each digit.

Resources

- Number Cards 0–9 (page 136)
- “Place Value Grid” (page 19)
- Pre-Test/Post-Test (page 20)



Mental Math and Number Sense Skills

Warm-Up Activity

1. With the class, practice counting forward and backward by tens from a three-, four-, or five-digit number.
2. Provide an example number, such as 157, and ask students to count forward together, by tens, to 257.
3. Provide an example number, such as 1,734, and ask students to count backward together, by tens, to 1,614.
4. Continue assigning starting numbers and ask students to count forward and backward in tens. In each case, give the class an appropriate ending number.



Whole-Class Skills Lesson

If the Pre-Test (page 20) has not yet been given, administer it at this time. (See pages 8–11 for testing information.)

Prepare to group students for the Differentiated-Group Skills Practice which follows the Whole-Class Lesson.

Use the directions below, or the PowerPoint® presentation (see PowerPoint® Lesson Option preview, page 18), to teach this lesson.

1. Tell students, “Today we are going to identify the place value of digits in large numbers.”
2. On the board or overhead, draw a seven-column table and label each column as shown below.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
			2	4	5	6

3. Write a number in the grid with seven digits or less, and ask students to identify the place value of each digit as you write it. For example, 2,456 would be two thousands, four hundreds, five tens, and six ones.
4. Ask students to write the same number in words. For example, 2,456 would be two thousand, four hundred, fifty-six.
5. Repeat this for several other numbers, varying the number of digits.



Differentiated-Group Skills Practice



On Level – Teacher Directed

- Tell students a four-digit number, and instruct them to write it on the “Place Value Grid”. Have them identify what each digit represents, say the number, and write it in words at the bottom. Repeat this process with other examples.
- Give students problems that require them to identify the missing number in a number sentence as follows: $57,624 = 50,000 + \underline{\hspace{2cm}} + 600 + 20 + 4$



Below Level – Student Directed

- Instruct students to work with a partner and choose four, number cards 0–9. Tell them to arrange these to make the largest possible number, write it on “Place Value Grid”, and say what each digit represents.
- Have students write out the number in words in the space provided. Ask students to repeat the process, selecting four different cards each time.



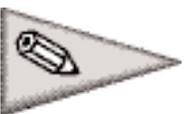
Above Level – Student Directed

- Tell students to choose five number cards 0–9 and arrange them so that they make the largest possible number. Tell them to write the number in words.
- Have students rearrange the same five cards to make the smallest number possible and write that number in words. (If desired, the activity can be extended to include six or seven cards.)

Lesson Review

Have students from each group take turns writing numbers on the board.

Ask the class what each digit represents, have them say the number aloud, and let a volunteer write it in words on the board.



Administer the Post-Test (page 20) at this time. (See pages 8–11 for testing information.)

Extension

Ask students, “Using all of the number cards 0–9 only once, what is the largest (or smallest) number that can be made? What is the sum of these numbers? What is the difference between these numbers? What do you notice?”

Additional Extension Resources:

- TRG Practices: 4–8, 18, 20

21-000774 Lesson 1.1

Today's Lesson

Identifying Place Value



21-000774 Lesson 1.1

Remember to count in tens together.
Here are the starting numbers.

316 **628**

7,592 **9,214**

63,425 **96,342**



21-000774 Lesson 1.1

Whole-Class Skills Lesson

Today we are going to identify the place value of digits in large numbers.



21-000774 Lesson 1.1

M	HTh	TTh	Th	H	T	O
9	7	4	1	6	5	0
			2	4	5	6
		3	8	2	1	9
	8	1	7	2	3	5

What does each digit represent?



21-000774 Lesson 1.1

Group Practice

 Practice writing four-digit numbers in words.

 Choose 4, 5, or 6 cards and arrange them to make the largest number possible. Write the number in words, and rearrange and repeat with a smaller number.



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Place Value Grid

1.1

Name _____ Date _____

Directions: Write numbers in the grid below. Place the digits in the correct columns and then identify what each represents. For example: 2,653 = 2 thousands, 6 hundreds, 5 tens, and 3 ones.

	Ten Thousands	Thousands	Hundreds	Tens	Ones
1.					
2.					
3.					
4.					
5.					
6.					

Directions: On the lines below, write each of the above numbers in words. (For example, 2,653 = two thousand, six hundred fifty-three.)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____



Pre-Test

Post-Test



1 Which number means eight ten-thousands, seven thousands, five hundreds, six tens, and nine ones?

- A 8,756
- B 87,659
- C 875,609
- D 87,569

1 Mount Everest is twenty-nine thousand, thirty-five feet tall. Which number shows this height?

- A 2,935
- B 29,350
- C 29,035
- D 290,035

2 What is the value of 6 in 906,415?

- F 6 ten thousands
- G 6 millions
- H 6 thousands
- J 6 hundreds

2 What is the value of the 6 in 8,672,913?

- F 6 hundreds
- G 6 ten thousands
- H 6 millions
- J 6 hundred thousands

3 Franklin School has 1,287 students. How is the number read?

- A one thousand, two hundred eighty-seven
- B ten thousand, two hundred eighty-seven
- C twelve hundred seventy
- D one hundred thousand, eighty-seven

3 What number means seven millions, no hundred thousands, four ten thousands, three thousands, no hundreds, no tens, and five ones?

- A 704,315
- B 7,043,050
- C 7,043,005
- D 7,430,005

Name

3

Name

3

Problem-Solving Strategies

Open-Ended Problem Solving

Open-ended problems challenge a learner's thinking. In looking at open-ended problems, students explore problems that may be answered in a number of ways. However, these problems should always be accurately computed.

These problem-solving activities are vitally important, and assist teachers to gauge the level at which their students are achieving, because students respond to challenges at their own level of development. The process that is used is more important than simply achieving the answer. When structuring problems, words such as create, make, design, investigate, and explore should be used.

Ask students to develop their own open-ended problems. It is important that students develop their own open-ended problems and solutions as this involves them in developing their thinking and phrasing. If you are helping students solve problems they have set up, model your work as you solve them, as students will learn from your approach to their problems.

There are some specific skills and strategies that are useful when working with this approach.

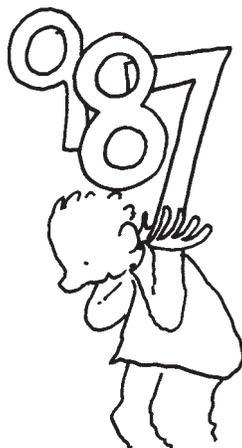
Using Numbered or Labeled Counters

When problems become involved, using numbered or labeled counters can help students visualize a problem and its solutions. The counters can be easily manipulated and altered and other combinations found. Changes can easily be made if work is incorrect. When the correct solution is found, it can be written down.

Trying Different Combinations of Numbers

For example, if attempting to solve a problem in which you are to determine the largest product possible using five different numbers, start by using the largest three-digit number and then the largest two-digit number to see the different answers.

(See also Teaching Example 1.)



Using the largest three-digit number:

$$\begin{array}{r} 987 \\ \times 65 \\ \hline 64,155 \end{array}$$

Using the largest two-digit number:

$$\begin{array}{r} 642 \\ \times 98 \\ \hline 62,916 \end{array}$$

Continue Working to Find as Many Solutions as Possible

This involves students in working to find more than one answer, and manipulating figures in order to look at them differently. Students are thereby encouraged to become involved with genuine mathematical solutions.

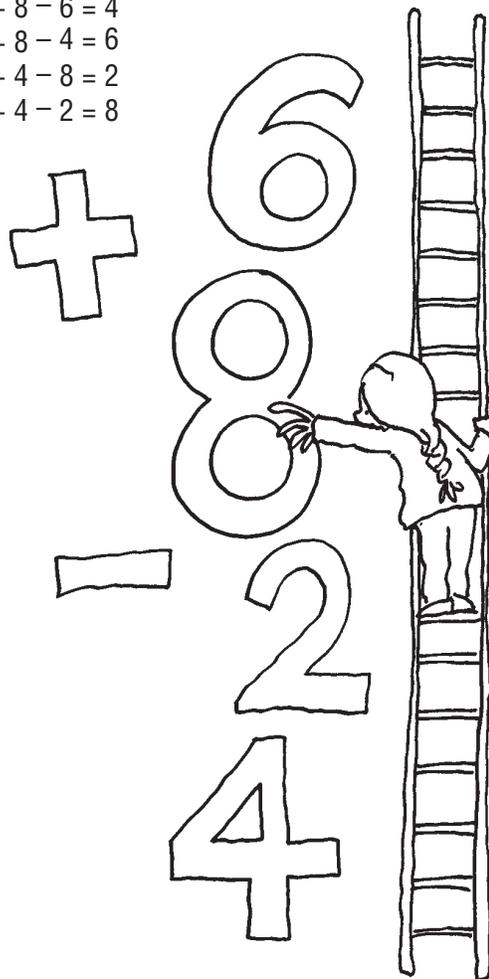
Example: What can we do with 2, 4, 6, 8, =, +, and -?

$$2 + 8 - 6 = 4$$

$$2 + 8 - 4 = 6$$

$$6 + 4 - 8 = 2$$

$$6 + 4 - 2 = 8$$



Problem-Solving Lesson

Using Strategies to Solve Real-Life Problems

Resources

- ✓ Problem-Solving Strategy Card and transparencies: “Looking for a Pattern”
- ✓ Real-Life Problem-Solving Card and transparencies: “Going to the Movies”
- ✓ Optional: copies of strategy and problem-solving cards (see Teacher Resource Guide CD-ROM)
If an overhead projector is unavailable, use the problem-solving card provided (or download from the CD-ROM) to prepare copies of the cards for students to use during the lesson.

Introducing the Strategy

Problem-Solving Strategy Card: Looking for a Pattern

Introduce the Strategy

Use the strategy card to introduce or review the strategy.

1. Place the transparency for Side A of the strategy card on the overhead projector. Discuss the information in the box, reinforcing reasons why looking for a pattern can be a useful tool for solving problems.
2. Discuss the rest of the information on the page. After working through the problem “Strawberry Picking,” ask students to explain how looking for a pattern helped them determine the answer. Point out how finding relationships between numbers helps to find a pattern.
3. Place the transparency for Side B of the strategy card on the overhead projector. Read and discuss the Sample Problem.
4. Read and discuss each step (Understanding the Problem, Planning and Communicating a Solution, Reflecting and Generalizing, and Extension) as it applies to the Sample Problem. Reinforce with students the importance of reflecting on how the solution was reached. Also, point out that they should think about other problems that can be created as an extension of the original one.

Review the Strategy

1. Reread the strategy card with the students and review key information. Remind students that when using this strategy, they are looking for patterns to help find a solution to a problem.
2. Review with students how to compare numbers in order to look at relationships. If the information is not already organized, it may be helpful to see patterns more readily if the information is placed in a table or an organized list. By comparing information within the table or list, students may be able to see a pattern develop.
3. In order to use looking for a pattern as a strategy to solve a problem, students will need to understand the question being asked. What information are you going to be determining?
4. Read the problem to determine the information already known. Suggest that underlining, highlighting, or making a quick list of the information from the problem may help when finding the solution.

Problem-Solving Lesson

Using Strategies to Solve Real-Life Problems

Applying the Strategy

Real-Life Problem-Solving Card: “Going to the Movies”

UNDERSTANDING THE PROBLEM

Getting Ready

- On an overhead projector, place the transparency for Side A of the “Going to the Movies” card.
- Together, read and discuss the information on Side A of “Going to the Movies.” Ask students what kind of movies they like to see. Who takes them to the movies? Do they get any snacks from the concession stand when they go to the movies? Discuss any other information that students may need clarified before solving the problems.
- Explain to students that they will be assigned to a group and given a specific problem to solve based on the information on Side A of “Going to the Movies.” They will use the strategy of looking for a pattern to solve the problem.
- Divide the class into groups based on ability levels indicated on Side B of the card (Problem A: Below Level, Problem B: On Level, Problem C: Above Level).

What do I know?

- Help students identify information that will be needed to solve the problem. Students may have to reread the passage in order to locate some of the needed information. Also, as the problem is reread, help them identify key words that may indicate what operation(s) will be needed in order to solve the problem.

Problem A: A small popcorn costs \$4.50, a medium popcorn costs \$5.25, and a large popcorn costs \$6.00.

Problem B: A small popcorn costs \$4.50, a medium popcorn costs \$5.25, and a large popcorn costs \$6.00. A small drink costs \$3.75, a medium drink costs \$4.00, and a large drink costs \$4.25.

Problem C: A small popcorn costs \$4.50, a medium popcorn costs \$5.25, and a large popcorn costs \$6.00. A small drink costs \$3.75, a medium drink costs \$4.00, and a large drink costs \$4.25. A child’s ticket costs \$6.50. You have \$20.00

What do I need to find out?

- Identify what question needs to be answered when solving the problem.
- Have students write the question(s) that needs to be answered at the top of their work paper.

Problem A: How much would it cost for an extra large popcorn?

Problem B: How much would it cost for an extra large popcorn and an extra large soda?

Problem C: How much does an extra large popcorn cost? How much does an extra large drink cost? What is the total cost of the popcorn, drink, and ticket? How much money is left over?

Problem-Solving Lesson

Using Strategies to Solve Real-Life Problems

Applying the Strategy (cont.)

Real-Life Problem-Solving Card: “Going to the Movies” (cont.)

PLANNING AND COMMUNICATING A SOLUTION

Solving the Problem

Use the following information to guide students’ thinking as they plan and solve the specific problem to which they are assigned.

For all three of these problems, have students draw a table with three columns and four rows. Across the top, create headings for sizes, popcorn, and soda. Under the sizes heading, list S, M, L to label the rows. Then, complete the table with the information known from the problem.

Problem A

Have students use the table to look for a pattern. Determine the difference in cost between a medium and a small popcorn. Determine the difference in cost between the large and the medium popcorn. Look for a pattern. Then, extend the pattern by adding the difference to the cost of a large popcorn to determine the cost of an extra large popcorn.

Problem B

Have students use the table to look for a pattern. Determine the difference in cost between a medium and a small popcorn. Determine the difference in cost between the large and the medium popcorn. Look for a pattern. Then, do the same for the sizes of soda. Then, extend the pattern by adding the differences in costs to the cost of a large popcorn and the cost of a large soda. Have students add the cost of an extra large popcorn and soda together.

Problem C

Have students use the table to look for a pattern. Determine the difference in cost between a medium and a small popcorn. Determine the difference in cost between the large and the medium popcorn. Look for a pattern. Then, extend the pattern by adding the difference in cost to the cost of a large popcorn. Follow the same procedure for determining the cost of an extra large drink. Then, look for the cost of the movie ticket. Add the cost of each of the items purchased to determine the total cost. Students will then subtract the total cost from \$20.00 to determine how much money is left over.

Solutions

Problem A

Find the difference between a medium popcorn and a small popcorn by subtracting. ($\$5.25 - \$4.50 = \$0.75$) Then, find the difference between a large popcorn and a medium popcorn. ($\$6.00 - \$5.25 = \$0.75$) Since there is \$0.75 difference between each of the sizes, the size of an extra large popcorn can be determined by adding \$0.75 to the cost of a large popcorn. ($\$6.00 + \$0.75 = \$6.75$) An extra large popcorn would cost \$6.75.

Problem B

See Problem A for the solution to finding the cost of an extra large popcorn.

Follow a similar procedure for finding the cost of an extra large drink. Find the difference between a medium drink and a small drink by subtracting. ($\$4.00 - \$3.75 = \$0.25$) Then, find the difference between a large drink and a medium drink. ($\$4.25 - \$4.00 = \$0.25$) Since there is \$0.25 difference between each of the sizes, the size of an extra large drink can be determined by

Problem-Solving Lesson

Using Strategies to Solve Real-Life Problems

Applying the Strategy (cont.)

Real-Life Problem-Solving Card: “Going to the Movies” (cont.)

adding \$0.25 to the cost of a large drink. ($\$4.25 + \$0.25 = \$4.50$) An extra large drink would cost \$4.50

Add the cost of an extra large popcorn and an extra large soda together. ($\$6.75 + \$4.50 = \$11.25$) An extra large popcorn and an extra large soda would cost \$11.25.

Problem C

See Problem A for the solution to finding the cost of an extra large popcorn. See Problem B for finding the cost of an extra large soda.

To determine the total amount spent, add the amount of a child’s ticket, \$6.50, the amount of an extra large popcorn, \$6.75, and the amount of an extra large drink \$4.50. ($\$6.75 + \$6.75 + \$4.50 = \17.75)

To determine how much money is left, subtract the amount spent from the amount with which you started. ($\$20.00 - \$17.75 = \$2.25$) You would have \$2.25 left.

REFLECTING AND GENERALIZING

Problem A

Students can check their answers using addition. By adding \$0.75 to the cost of each size of popcorn, the answer should be the cost of the next size of popcorn. A small popcorn costs \$4.50. ($\$4.50 + \$0.75 = \5.25) A medium popcorn costs \$5.25. Add \$0.75 to the cost of a medium popcorn. ($\$5.25 + \$0.75 = \$6.00$) A large popcorn costs \$6.00. Add \$0.75 to the cost of a large popcorn. ($\$6.00 + \$0.75 = \$6.75$) An extra large popcorn would cost \$6.75. The answer makes sense.

Problem B

See Problem A for a way to check the cost of the extra large popcorn.

Students can check the cost of the extra large drink by adding \$0.25 to the cost of each size of drink. The answer should be the cost of the next size of drink. A small drink costs \$3.75. ($\$3.75 + \$0.25 = \4.00) A medium drink does cost \$4.00. Add \$0.25 to the cost of a medium drink. ($\$4.00 + \$0.25 = \$4.25$) A large drink does cost \$4.25. Add \$0.25 to the cost of a large drink. ($\$4.25 + \$0.25 = \$4.50$) An extra drink would cost \$4.50. The answer makes sense.

Subtract the cost of an extra large popcorn from the total cost determined, $\$11.25 - \$6.75 = \$4.50$ \$4.50 is the cost of an extra large soda. The answer makes sense.

Problem C

See Problem A and B for ways to check the cost of the extra large popcorn and extra large soda.

Check the total amount spent by subtracting. Subtract the cost of a ticket from the total amount spent. $\$17.75 - \$6.50 = \$11.00$. Then, subtract the cost of an extra large drink. $\$11.00 - \$4.50 = \$6.50$ The amount left, \$6.50 is the cost of an extra large popcorn. The problem checks.

To check the amount left, add the amount left to the total amount spent. $\$2.25 + \$17.75 = \$20.00$ This is the total amount you had before you went to the movies. This answer makes sense.

EXTENSION

Use the Class Challenge to solve an additional problem related to the topic. (Answers will vary depending on the size of the class and the size choice of snacks.)

Going to the Movies

With annual ticket sales of over 7.5 billion dollars, going to the movies is a favorite activity for all ages.

Many people attribute the first movie to Eadweard Muybridge. In 1877, Mr. Muybridge took seven pictures of a horse running. He projected the pictures on a screen, using a lantern. By changing the images quickly, it appeared as if the horse was running. The first full-length feature film was made

in 1913 by Cecil B. DeMille. Until then, movies were much shorter in length. Early movies were silent and were in black and white. It was not until the late 1920s that sound was added. The development of a new camera in the early 1930s allowed movies to be filmed in color.

Movie technology has advanced a lot since then. Most movies are now being made digitally. Computers are used for special effects, animation, editing, and sound design. This technology allows film-makers to easily manipulate color, lighting, and sound to improve the overall effect of a movie. In fact, some animated movies, such as *Toy Story*, are made completely on the computer.

A movie is said to be a blockbuster if it is a popular movie that makes a lot of money. Some of the highest grossing blockbusters of all time include *Jurassic Park* (\$920 million), *Spiderman* (\$806 million), and *Star Wars* (\$798 million).



The first movie theater experience began in 1895 when people watched a movie in the basement of a café. Today, many megaplex movie theaters have multiple screens. Surround sound and stadium seating complete the movie-going experience.

For many people, a trip to the movie theater would not be complete without a stop at the concession stand. Candy, hot dogs, soda, and pretzels are some of the food items moviegoers can purchase to snack on while viewing the movie. But the most popular item at the concession stand is popcorn. Popcorn was first sold at movie theaters in 1930. Today, it is the best selling item at the concession stand. In 1955, popcorn sales at movie theaters totaled \$355 million. Today, movie theaters in the United States sell \$15 billion in popcorn each year. That figure accounts for about 30% of all of the popcorn sold in the United States.

Going to the Movies

WHAT IS THE PROBLEM?

You are going to the movie theater to see the latest blockbuster movie. You will have to determine how much it will cost for your evening at the movies.

Use the information on the other side of the card and below to answer the questions. You will be asked to work on one of the problems below. Before you begin solving the problems, be sure to locate the information you will need.

Think about how you compare information in order to look for a pattern.

Remember: If you have trouble planning ways to solve the problem, reread it and rethink your strategy. When you have a solution, think about whether or not your answer makes sense.

Ticket Prices	
Adult ticket	\$9.50
Child ticket	\$6.50
Seniors	\$7.50
Concession Stand Menu	
Small popcorn	\$4.50
Medium popcorn	\$5.25
Large popcorn	\$6.00
Small soda	\$3.75
Medium soda	\$4.00
Large soda	\$4.25

PROBLEM-SOLVING STRATEGY: LOOKING FOR A PATTERN

Problem A

The movie theater also offers an extra large popcorn. How much would it cost you for an extra large popcorn?

Hint: Find the differences in cost between a small and medium and a medium and large. Create a table in order to look for a pattern.

Problem B

The movie theater also offers extra large portions. How much would it cost you for an extra large popcorn and an extra large soda?

Hint: Find the differences in cost between a small and medium and a medium and large. Create a table in order to look for a pattern.

Problem C

The movie theater also offers extra large portions. You go to the movie theater and purchase an extra large popcorn and an extra large drink. How much will you spend, including the cost of the ticket? If you pay with a \$20.00 bill, how much money will you have left?

Hint: Find the differences in cost between a small and medium and a medium and large. Create a table in order to look for a pattern.

Class Challenge
Plan a class trip to the movies. How much would it cost for tickets for the entire class to see a movie? If everyone in the class got the same size popcorn and drink, what would the total cost at the concession stand be?

Looking for a Pattern

Mathematical patterns can be found everywhere—in nature, in numbers, and in shapes, for example. So, it is not surprising that the strategy of “Looking for a Pattern” is used often.

When you find a pattern, it becomes easy to predict what comes next.

You will find that the problem-solving strategies you learn are sometimes

connected to one another. You may use more than one strategy to solve a problem. As you learn more about “Looking for a Pattern,” you will find that it is often an extension of “Drawing a Table” or “Creating an Organized List.”

Read the information and examples below to learn the strategy of looking for patterns to solve problems.

Finding a Pattern in a Table

With some problems, you may need to create a table before you can look for a pattern.

You need to decide if there are one, two, three or more things that change (variables) in the pattern. Is a “total” column necessary?

Problem: Strawberry Picking

- When Janie went strawberry picking, one out of every six strawberries had worm holes.

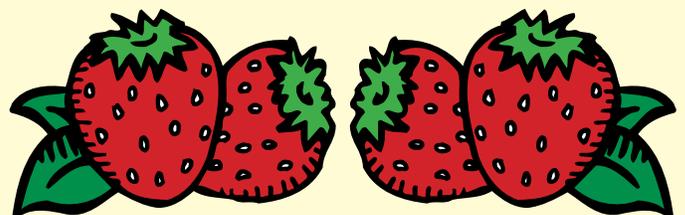
How many good strawberries were there out of 84?

Solve the problem by drawing a table with three columns labeled “Good”, “Bad”, and “Number of Strawberries.”

You can see the pattern in the table below. The “Good” column is increasing in multiples of 5 and the “Bad” column is increasing by 1.

If a total of 84 strawberries are picked, 70 will be good and 14 will be bad.

Good	Bad	Number of Strawberries
5	1	6
10	2	12
15	3	18
20	4	24
25	5	30
30	6	36
35	7	42



Using the Strategy

Sample Problem

The Ross Family is working on a fitness program. On the first day, they cycle around the track three times. On the second day, they cycle seven times. On the third day, they cycle eleven times.

How many days must the Ross family exercise before they reach their goal of cycling around the track 31 times?

UNDERSTANDING THE PROBLEM

What do I know?

The number of cycles around the track increases each day.

What do I need to find out?

How many days the Ross family must exercise before they reach their goal.

PLANNING AND COMMUNICATING A SOLUTION

As you can see from the problem, the number of cycles around the track is increasing each day.

You can draw a table, like the one below, to see if there is a pattern.

Days	Cycles
1	3
2	7
3	11
4	15
5	19
6	23
7	27
8	31

Look at the table to find the pattern and solution.

It will take 8 days for the Ross family to reach their goal of 31 times around the track.

REFLECTING AND GENERALIZING

Once the table has been drawn, it is easy to see a pattern. The numbers are increasing by four cycles a day.

EXTENSION

Here is a possible way to make a new problem. Instead of increasing by the same number of cycles each day, the number of cycles per day could increase.

For example, the first day the number of cycles is set at one, the second day the number increases by two, the third day the number of cycles increases by three, and so on.



Introduction

The Skill Application Games

☐ Reinforcing Skills

A set of 14 games is provided. These full-color cards are ideal for centers and small-group interaction. The cards are coordinated with the lesson units. They reinforce several skills taught within each unit. Many cards are designed to make math content “real” to students by providing a real-life setting, or a content area connection. Each card includes student directions as well as reproducible page information. For information about the games, see the Management and Skill Application Games sections of this guide.

The CD-ROM

☐ A Source for Printable Materials

A CD-ROM is included in the back of this guide. It contains the following materials:

- Placement Tests from Units 1–7
- Pre-Test and Post-Test pages from Units 1–7
- Student Activity Pages from Units 1–7
- Skills Application Games (student directions and related reproducible information)
- Problem-Solving Strategy Cards
- Real-Life Problem-Solving Cards
- PowerPoint® Lesson Options (slide presentations) for all unit lessons (The PowerPoint® lesson offers an alternative method for teaching the Warm-Up Activity and the Whole-Class Skills Lesson, as well as a listing Differentiated-Group Practice activities of the lesson.)
- Reproducibles from the last section of each unit
- Additional Practice Pages (Over 100 pages of student practice to reinforce basic skills are introduced in each of the units. Answer keys are included.)
- Glossary from Teacher Resource Guide

Note: The printable pages on this CD allow you to download and print copies of the materials listed above. Simple directions for using the CD are presented in the READ ME files.

Management

⇒ Pacing Plans

The Exploring Math kit is designed for flexibility, especially when used in summer intervention or after-school programs where sessions are limited.

Below are some suggestions for using the kit for different purposes and/or in different time frames.

Special Programs—Summer School/Limited Session

Daily Eight-Week Program (40 class sessions)

Over a period of two to three sessions, administer and score the Placement Test to determine which units to use throughout the eight-week session. Rank these units according to the overall class score for each (See Placement Test Class Record, pages 71 and 72.).

To plan the number of sessions per unit, divide the remaining sessions (about 37) by the number of units. You may wish to adjust this number so that more sessions are devoted to the unit on which the students scored the lowest.

Option A: Target Lessons from 2 to 4 Units

Sample Pacing Chart: Daily Eight-Week Program (3 Units)								
Week								
Day	1	2	3	4	5	6	7	8
Mon	Placement Test	Unit 2 L-1.4	Unit 2 L-1.12	Unit 2 L-1.20	Unit 4 L-1.4	Unit 4 L-1.11	Unit 5 L-1.2	Unit 5 L-1.9
Tues	Placement Test	Unit 2 L-1.6	Unit 2 L-1.13	Unit 2 L-1.21	Unit 4 L-1.6	Unit 4 L-1.13	Unit 5 L-1.3	Unit 5 L-1.11
Wed	Unit 2 L-1.1	Unit 2 L-1.8	Unit 2 L-1.15	Unit 4 L-1.1	Unit 4 L-1.7	Unit 4 L-1.14	Unit 5 L-1.5	Unit 5 L-1.12
Thurs	Unit 2 L-1.2	Unit 2 L-1.9	Unit 2 L-1.18	Unit 4 L-1.2	Unit 4 L-1.9	Unit 4 L-1.16	Unit 5 L-1.6	Unit 5 L-1.13
Fri	Unit 2 L-1.3	Unit 2 L-1.11	Unit 2 L-1.19	Unit 4 L-1.3	Unit 4 L-1.10	Unit 4 L-1.18	Unit 5 L-1.8	Unit 5 L-1.15

This sample pacing chart was developed after the Placement Test results indicated that units 2, 4, and 5 should be covered in the eight-week session. Since skills and concepts from Unit 2 need the most attention, more sessions were scheduled to teach lessons from this unit.

Management

⇒ Pacing Plans (cont.)

Daily Eight-Week Program (40 class sessions)

Option B: Teach One Unit

Administer the Placement Test as suggested in Option A. If only one unit is to be completed during this time, you will have an opportunity to cover some lessons over a two-day period. This gives you time to provide additional practice on difficult skills and to investigate the Extension activities more thoroughly. You should also plan more problem-solving practice time, using the strategies learned in the unit.

Option C: Choose Lessons from All Units

Administer the Placement Test as suggested in Option A. If the curriculum guidelines from your intervention program (and/or the Placement Test scores) indicate that all topics should be covered, it will be necessary to reduce the number of lessons offered for each unit. In an eight-week session, for example, you could teach four to five lessons per week, allowing five or six sessions for problem-solving practice. Since each unit contains an average of 20 lessons, you will need to decide which lessons to choose from each unit. (The unit pre-tests can serve as an indicator.)

Daily Six-Week and Four-Week Programs

In a six-week session, with 30 class periods, it is recommended that you select no more than three units on which to focus. This allows you to complete about eight lessons per unit, with remaining sessions devoted to the initial testing and the problem-solving practices.

Four-week programs provide you with 20 class periods. Since time is limited, your students' needs would be best served if you plan to cover two units at the most.

Whether your program is four weeks or six weeks long, the pacing plan can be similar to the sample daily eight-week program, with adjustments made to cover three-fourths or one-half of the material scheduled for eight weeks.

Ongoing Regular and After-School Programs

Exploring Math was designed as an intervention and reinforcement program. The lessons in the kit cover essential topics that are taught throughout the regular school year. The lessons can support your existing core mathematics program (based on a 40-week time frame, with a minimum of 2.5 to 4 hours of mathematics instruction per week). As you develop a pacing plan for your existing program, include related lessons from this kit that can be used (during or after school) as reinforcement for specific groups of students with diagnosed needs.

Management

⇒ Pacing Plans (cont.)

Note: Depending on the length of each day's session, it is possible to complete two related lessons in one day. Most lessons take between 50 and 60 minutes to complete. If your summer school, or other limited-session program, has daily sessions of 2 to 2 1/2 hours, it is possible to double up on the lessons. (Adjust pre- and post-testing accordingly.)

Problem-Solving Practice: If possible, provide as much problem-solving practice as possible during the sessions. Since Problem-Solving Lessons are provided in each unit, students will be exposed to at least two opportunities to use specific strategies and organized methods for solving problems. Once students learn how to apply a strategy, they should not be limited to the lessons in the unit. If possible, allow some time each week to solve a few problems using a particular strategy.

Additional Practice: Where appropriate, assign homework from the Extension activities in the lessons, or from selected practice pages on the teacher's guide CD-ROM.

Summary

Whether you are planning an intervention program for the summer, or working within a regular or after-school program, use the following "3 D's" to organize your daily schedule.

- Diagnose areas of greatest need by administering and scoring the Placement Test to determine your students' needs.
- Decide which units (and the number of lessons) to include within the specific time frame of your program.
- Develop a pacing chart to plan the specific lessons and suggested activities prescribed in this kit.