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**Appendices**
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Research on the Effectiveness of Intervention

For many people, “Math is right up there with snakes, public speaking, and heights” (Burns 1998). Mention the word mathematics and the room grows silent, people squirm in their seats, and small beads of sweat appear on their temples. In an effort to keep others from the disappointment that will certainly ensue, a disclaimer is spoken, “I am not good at math.” Parents hire tutors because they do not feel comfortable helping their fifth graders with math homework. People hail the mathematics professor as a god-like figure. These behaviors certainly can discourage both learners and teachers alike. Even so, mathematics cannot be ignored because it is relevant every day in real life. In fact, it might be nearly impossible to live a day without math. Counting spare coins in a jar, estimating the cost of an item to include tax, and converting recipes for larger groups are just a few of the ways mathematics enters people’s daily routines. Because of this, it is imperative that this mathematics phobia be put to rest. But how can these misconceptions about mathematics be overcome? The cure for mathematics phobia might be as simple as providing students with the necessary skills and opportunities to be successful in mathematics.

The Need for Intervention

The National Council for Teachers of Mathematics (NCTM) has some high expectations or goals for students. They want students to become mathematical problem solvers, learn to communicate and reason mathematically, and make connections (NCTM 2000). The task of teaching mathematics in classrooms today appears more difficult because today’s educators understand that curriculum must be differentiated to meet the needs of all students. Students come into mathematics class with different levels of readiness, learning styles, and interests. To meet the needs of all learners, teachers must provide varied levels of time, structure, support, and complexity with different intensities for these learners (Coleman 2003). Targeted Mathematics Intervention incorporates differentiated activities throughout the lessons. The differentiation is seen in the focus on vocabulary, small-group guided practice, and open-ended activities.

The learning differences among students can be illustrated in several ways. The basic mathematics facts require some memorization. Some students have not been taught memorization skills. English language learners have language and communication issues, and this affects every content area, including math. Students who have processing difficulties find the mathematics symbols and numerals confusing. Other students perform poorly in mathematics simply because they have low self-esteem and short attention spans. Some students have even developed the attitude that mathematics is scary or boring based on previous experiences. Personality dictates that some students are passive learners or are simply disorganized. This makes the task of teaching mathematics complicated for teachers. The one-size-fits-all curriculum and instruction cannot possibly accomplish the goal of reaching all students (Tomlinson 2003). For these reasons, mathematics intervention is necessary. Still, the question remains: What kinds of mathematics interventions work?
Differentiating Student Guided Practice

Response to Intervention in the Mathematics Classroom

All students learn differently and struggle with different mathematical concepts. In one classroom, teachers can have students who are above grade level, on grade level, below grade level, and English language learners. Not all below-grade-level students struggle in the same areas and fit into the same “category.” Because of this, many of the same researchers who created the Reading First initiative developed a system of identification known as Response to Intervention (RTI). The RTI model supports the idea that teachers should look for curricular intervention designed to bring a child back up to speed as soon as he or she begins having difficulties. “RTI has the potential then to allow disabilities to be identified and defined based on the response a child has to the interventions that are tried” (Cruey 2006). Depending on the levels of difficulty they are having with the mathematics curriculum, students are classified as Tier 1, 2, or 3. Specific definitions of these tiers differ from state to state, but the following are general descriptions.

**Tier 1 students** are generally making good progress toward the standards, but may be experiencing temporary or minor difficulties. These students may struggle only in a few of the overall areas of mathematical concepts. They usually benefit from peer work and parental involvement. They would also benefit from confidence boosters when they are succeeding. Although they are moving ahead, any problems that do arise should be diagnosed and addressed quickly in order to ensure that these students continue to succeed and do not fall behind.

**Tier 2 students** may be one or two standard deviations below the mean on standardized tests. These students are struggling in various areas and these struggles are affecting their overall success in a mathematics classroom. These students can usually respond to in-class differentiation strategies and do not often need the help of student study teams.

**Tier 3 students** are seriously at risk of failing to meet the standards as indicated by their extremely and chronically low performance on one or more measures of the standardized test. These students are often the ones who are being analyzed by some type of in-house student assistance team in order to look for overall interventions and solutions. In the classroom, these would be the students who are having difficulties in most of the assignments and failing most of the assessments.

Differentiation by Specific Needs

**Below-Grade-Level Students**
Below-grade-level students will probably need concepts to be made more concrete for them. They may also need extra work with manipulatives and application games. By giving them extra support and understanding, these students will feel more secure and have greater success.

- Allow partner work for oral rehearsal of solutions and allocate extra time for guided practice.
- Allow for kinesthetic activities where they organize the step-by-step processes on flash cards before they actually use the information to solve problems.
- Have easy-to-follow notes of the most important procedural information already made up for these students to add to.
Program Overview (cont.)

Planning for Intervention

*Targeted Mathematics Intervention* can be used in multiple ways according to district requirements, school resources, and student needs. The program has four main goals that make it flexible for various intervention programs. Each goal is summarized below:

**Targeted Instruction of Key Content Standards**—This 30-lesson program can be used in a variety of settings to help bring students up to grade level or prepare them for standardized tests. The chosen standards are targeted for each grade level to hit on the most important mathematical concepts. The lessons are strong in concrete examples, multiple representations, and the use of various algorithms.

**Different from the Regular Classroom Curriculum**—Students in mathematics intervention programs do not need their curriculum from the regular classroom to be repeated. They need more engaging curriculum to grab and keep their interest. Furthermore, fun activities such as games and hands-on lessons help all types of learners.

**Easy to Use**—Teachers in mathematics intervention programs are not always mathematics teachers. Many times, teachers are teaching off grade level or out of their content area. So, this program includes all necessary materials and has straightforward lessons with clear teacher directions. And yet, the lessons are flexible enough that experienced teachers can incorporate their own teaching styles and strategies.

**Compact and Portable**—Many intervention programs do not take place in well-stocked mathematics classrooms. This program is compact, yet it contains all the materials teachers need to be successful. Teachers can take the box and student books and successfully teach the 30 lessons. However, if a school has extra resources (manipulatives, computers, calculators, etc.), these resources can easily be tied into the program.

Pacing Plans

When planning the pacing of a curriculum program, educators need to analyze student data to determine standards on which to focus. This program has targeted the 30 most-tested standards for each grade level. These lessons can be taught one a day over six weeks. The *Six-Week Pacing Plan* (page 30) shows each two-hour lesson. However, many programs are not exactly six weeks long; so 20 key standards have been outlined in the *Sample Four-Week Pacing Plan* (page 31). This flexible program can also be used over longer periods of time. The *Sample After-School Pacing Plan* (page 32) shows how the 60 hours can be spread out over six months.

To further adapt the program to instructional time frames that are shorter than six weeks, it is highly recommended that teachers give the *Diagnostic Test* to determine which standards students have not mastered. Teachers can then use the *Diagnostic Test Item Analysis* (pages 37–38) to analyze their students’ results and select lessons to target. In addition, teachers can modify the lessons by using the suggested ways to accelerate the curriculum and decelerate the curriculum on the next page.
### Diagnostic Test

#### 1. 5 tens + 4 ones = ______
- A. 45
- B. 9
- C. 54
- D. 45

#### 4. Which number is the greatest?
- F. 75
- G. 63
- H. 57
- J. 38

#### 2. Which two-digit number CANNOT be made using the numbers 4, 7, and 9?
- F. 45
- G. 79
- H. 94
- J. 49

#### 5. Which picture is circled?
- A. ninth
- B. tenth
- C. eighth
- D. seventh

#### 3. Put these numbers in order from least to greatest.
- 12 11 23 21
- A. 11, 12, 21, 23
- B. 12, 11, 21, 23
- C. 11, 21, 12, 23
- D. 12, 21, 11, 23

#### 6. Use the number line to solve. 5 + 8 = ______
- F. 12
- H. 13
- G. 14
- J. 15
7. Yi picked 15 flowers. She gave 8 flowers to her mom. How many flowers does she have left?

Which number sentence is used to solve this problem?

A. $15 + 8 = 23$
B. $15 - 8 = 7$
C. $15 - 5 = 10$
D. $15 + 7 = 8$

10. Solve.

$8 + 6 - 3 = _____$

F. 14
G. 17
H. 3
J. 11

8. Use the number line to solve.

$22 - 4 = _____$

F. 17
G. 18
H. 19
J. 16

11. Which number completes the fact family?

$6 + 5 = 11 \quad 5 + 6 = 11$
$11 - 5 = 6 \quad 11 - ____ = 5$

A. 6
B. 5
C. 11
D. 16

9. If Bill eats 3 cupcakes from his batch, how many are left?

A. 10
B. 9
C. 8
D. 7

12. Which shows the number 12?

F. eleven
G. $10 + 5$
H. 1 ten and 2 ones
J. $15 - 2$
Investigating Tens and Ones

Materials
- Student Guided Practice Book
  - Tens and Ones (page 11; page011.pdf)
  - Acting It Out or Using Concrete Materials 1 (page 12; page012.pdf)
  - Standardized Test Preparation 1 (page 13; page013.pdf)
  - Spin It! Directions (page 136; page136.pdf)
- Punchouts folder
  - Base Ten Blocks (baseten.pdf)
  - Spin It! Cards (spinit.pdf)
  - Counters (counters.pdf)
- Transparency folder
  - Acting It Out or Using Concrete Materials 1 (trans01.pdf)
- PowerPoint folder on the CD
  - Investigating Ones and Tens (lesson01.ppt) (optional)
- Game board
  - Party Time
- four spinners
- paper and pencils

Learning Objective
- Count and group objects by ones and tens.

Warm-up Activity 15 min.
Skill: Counting
1. Settle students for the day and then ask the students to name a number greater than 20 that they know and can count up to.
2. Count out loud up to the highest number given.
3. Count backward from the number.
4. Choose a different number and count up to it, then backward from it.

Vocabulary 10 min.
Complete the Chart and Match (page 6) vocabulary activity using the words below. Definitions of these words are included on pages 240–245 in this book and in the Student Guided Practice Book.
- group
- greater than
- less than
- set
Investigating Tens and Ones (cont.)

Whole-Class Skills Lesson 25 min.

Use the directions below or the PowerPoint presentation to teach this lesson.

1. Tell students, “Today, we are going to put numbers into groups of tens and ones.”

2. Distribute Base Ten Blocks punchouts and a piece of paper to each student. Ask a student volunteer to say a number between 10 and 20. Have students collect the chosen number in unit squares on their pieces of paper, count them all out, and label the set with the correct number.

3. Model how to group unit squares into two sets (one set of ten, another in a set of ones) and direct students to follow your example. Ask students how many groups of ten there are and how many ones are remaining. For example, the number 15 would have one set of ten and one set of 5 ones.

4. Show students that one tens rod is the same as ten ones squares. Switch the set of ten ones squares for one tens rod and have students do the same. Reinforce that this representation still shows the original number. Write it as a whole number and in a T-chart. See the example below.

\[
\begin{array}{c|c}
15 & \\
\hline
1 & 5
\end{array}
\]

5. Ask a student volunteer to say a number between 20 and 30. Have students collect the chosen number in unit squares, count them all out, and label the set with the correct number.

6. As a group, split the unit squares into sets of ten. This time, there should be 2 sets of ten and a set of leftover unit squares. Ask students how many sets of ten and how many ones there are.

7. Ask students if any of the sets can be exchanged for tens rods. The students should know to switch the two sets of tens for 2 tens rods. Ask student volunteers to draw the numeric representation and the place value representation on the board or overhead. See the example below.

\[
\begin{array}{c|c}
27 & \\
\hline
2 & 7
\end{array}
\]

8. Ask a student volunteer to say a number between 50 and 60. Tell students that it would take a long time to count out that many unit squares. Help students count by tens to get to the chosen number. Every time the class says a multiple of ten, have students place a tens rod in front of them to represent each group of ten. Stop counting by tens when the class reaches 50. Then, count by ones to get to the chosen number. As the class counts by ones, have students place one unit square in front of them to represent the single units leading up to the chosen number.

\[
\begin{array}{c|c}
54 & \\
\hline
\text{Do:} & \\
\text{Say:} & 10 20 30 40 50 51 52 53 54
\end{array}
\]

10. Ask student volunteers to write the numeric and place value representations on the board or overhead. As time allows, repeat step 9 for other numbers greater than 50 and less than 100.

*Note: If you are using three-dimensional base 10 blocks, the correct terminology is as follows: unit cubes, tens rods, and hundreds flats.
Investigating Tens and Ones (cont.)

Differentiated Guided Practice 20 min.

● Below Level—Teacher Directed
- Distribute Base Ten Blocks punchouts to each student in the group.
- Using the manipulatives, count out the correct amount of tens rods and unit squares to represent each of the numbers on the Tens and Ones (SGPB page 11) activity sheet.
- Complete the activity sheet as a group.
- If time permits, divide the students into pairs. One student in the pair builds a number and the other counts the number and records it in written form. Then, the students switch roles.

■ On/Above Level—Student Directed
- Have students complete numbers 7–18 of Tens and Ones (SGPB page 11) individually.
- Give students the option of using Base Ten Blocks punchouts.
- Have students check their work in pairs. If students do not answer all questions correctly, instruct them to complete the rest of the activity sheet in pairs. If students answer all of the questions correctly, allow them to choose one number from the activity sheet and write that number in as many different forms as they can (e.g., word form, numeric form, tens and ones form, picture form, and number sentence form).

Problem Solving 20 min.
- Place the Problem-Solving Strategy Transparency: Acting It Out or Using Concrete Materials 1 on the overhead. Use the guiding callouts on page 48 to introduce the strategy to the students. A copy of this transparency is also included in the Student Guided Practice Book on page 12. Students can follow along and make notes as you review the transparency.

Test Preparation 10 min.
- Have students complete Standardized Test Preparation 1 in the Student Guided Practice Book (page 13). Give them about seven minutes. Then, have students trade papers and grade their work.

Learning Game 20 min.

Spin It!
- While students are completing the test preparation questions, set up four game stations. Each station needs one Party Time game board, a set of Spin It! Cards, and a handful of Counters.
- Review with students the Spin It! Directions (SGPB page 136). Answer any questions students have about how to play. You may want to model one round of play.
- Allow them time to play the game. Move among the students checking for understanding as they complete the mathematics problems. Make sure you stop the students with about five minutes left so that students can clean up the game stations.
LESSON 1

Investigating Tens and Ones (cont.)

Problem-Solving Transparency Callouts

Each student will need Counters punchouts and a piece of paper for this activity.

1. Read and discuss the information across the top of the page, reinforcing how acting out a problem using manipulatives, such as models, can be a useful tool for solving problems.

2. Read the problem aloud to determine the information already known. Have children brainstorm ways to find the solution.

3. Read and discuss each of the steps shown as it applies to the sample problem. Before reading the Planning and Communicating a Solution section, discuss with students how using concrete materials helps them find the solution to the problem. Have the students give suggestions for things to use to help solve the problem.

4. Read and discuss the problem with the students. Ask the students to think of ways to act out the problem.

5. Read and discuss each of the steps shown as it applies to the sample problem. Before reading the Planning and Communicating a Solution section, discuss with students how acting out the problem helps them find the solution. Explain to the students that they will be acting out this problem using pieces of paper.

6. Reinforce with students the importance of reflecting on how they found the solution. Ask students to explain how acting out the problem or using concrete materials helped them to see the solution.
Investigating Tens and Ones (cont.)

Student Pages and Punchouts Needed for the Lesson

Tens and Ones  
(SGPB page 11; page011.pdf)

Acting It Out or Using Concrete Materials 1  
(SGPB page 12; page012.pdf)

Standardized Test Preparation 1  
(SGPB page 13; page013.pdf)

Spin It! Directions  
(SGPB page 136; page136.pdf)

Base Ten Blocks  
(baseten.pdf)

Spin It! Cards  
(spinit.pdf)

Counters  
(counters.pdf)
Today’s Lesson
Investigating Tens and Ones

Warm-Up Activity
Today we will warm up today by counting.
Starting at 1, let’s count together.

Think of a number less than 20 to count up to.
Starting at 1, let’s count together.

Let’s count back from the same number until we get to 1.
Ready? Let’s go!

PowerPoint Presentation Slides

Whole-Class Skills Lesson
Today we are going to put numbers into groups of tens and ones.

Now let’s think of a number that is greater than 20 to count to.
Let’s count back from the same number until we get to 1.
Ready? Let’s go!

Today we are going to put numbers into groups of tens and ones.

Warm-Up Activity
Today we will warm up today by counting.
Starting at 1, let’s count together.

Let’s count back from the same number until we get to 1.
Ready? Let’s go!

Class: Count each piece of candy.

There are 18 pieces of candy in this group.
There is a set of 10 and a set of 8.
The 1 represents 1 set of ten.
The 8 represents the 8 ones that were left over.

There are 14 pieces of candy in this group.
There is a set of 10 and a set of 4.
The 1 represents 1 set of ten.
The 4 represents the 4 ones that were left over.

How does the set of 10 and the set of 8 relate to the number 18?

Now let’s try a larger number.
Let’s count sets of 10 and any left over.

How does the set of 10 and the set of 4 relate to the number 14?

Let’s count how many sets of 10 there are in all.
Now let’s count how many are left over.

There are 4 sets of 10.
There are 2 left over.
Name __________________________________________

Tens and Ones

Read how many tens and ones. Write the number on the line.

1. 2 tens 6 ones __________ 6. 9 tens 2 ones __________
2. 5 tens 4 ones __________ 7. 7 tens 5 ones __________
3. 3 tens 1 one ____________ 8. 1 ten 6 ones ____________
4. 8 tens 4 ones __________ 9. 4 tens 9 ones __________
5. 6 tens 8 ones __________ 10. 4 tens 7 ones __________

Draw a line to match the pictures with the tens □ and ones □.

11. 5 tens 2 ones
   a. □□□□□□

12. 3 tens 1 one
   b. □□□□□□□□□□

13. 7 tens 9 ones
   c. □□□□□□□□□□

14. 2 tens 5 ones
   d. □□□□□

15. 6 tens 3 ones
   e. □□□□□□□□□□

16. 4 tens 8 ones
   f. □□□□□□□□□□

17. 1 ten 6 ones
   g. □□□□□□□□

18. 4 tens 2 ones
   h. □□□□□□□□□□□□□□□□
Acting It Out or Using Concrete Materials 1

Sometimes acting out a problem will help find the solution. It is a fun way to see what is happening in the problem.

Counters and objects can be a big help too! They can be moved through the steps of a problem. This helps you keep track of what is happening as you solve it.

Read below to learn about how to use objects to solve problems. Read below to learn about when to act out problems.

Using Concrete Materials

Problem: Animal Crackers

The Problem
Caleb had 15 animal crackers. He ate some and had 6 left. How many animal crackers did Caleb eat?

Understanding the Problem
• What do I know?
  Caleb had 15 animal crackers. He ate some and had 6 left.

• What do I need to find out?
  How many animal crackers Caleb ate.

Planning and Communicating a Solution
Use counters or objects to find the answer. Place 15 counters in front of you. Take counters away until you have 6 left. Count how many you took away.

• Do you see the answer?
  Caleb ate 9 animal crackers.

Act it Out!

Problem: Folding Paper

The Problem
Ming is making a picture. She folds a piece of paper in half. Then she folds it in half again. How many rectangles does she have to draw in?

Understanding The Problem
• What do I know?
  Ming folds a paper in half two times.

• What do I need to find out?
  How many rectangles there are.

Planning and Communicating a Solution
Use a piece of paper to act out the answer. Fold the paper in half. Fold the paper in half again. Now, unfold the paper. Count the rectangles.

• Do you see the answer?
  There are four rectangles.

Reflecting and Generalizing
Using these strategies help find the answers more easily.
Name ________________________________

**Standardized Test Preparation 1**

1. Which picture shows 19?

   - A
   - B
   - C
   - D

2. Which fish is circled?

   - F fourth
   - G fifth
   - H sixth
   - J seventh

3. Tell how you got the answer to number one.

   ___________________________________________________________________
   ___________________________________________________________________