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Effective Math Instruction

Shifting to Meet Today's Standards

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Foreword by Ruth Harbin Miles

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Understanding the Standards

The Common Core State Standards in Mathematics (CCSS-M) are learning goals that outline what students should know and be able to do in the discipline of mathematics. The standards were created to ensure that all students are college and career ready when they graduate from high school, an effort coordinated by the Council of Chief State School Officers (CCSSO) and the National Governors' Association (NGA) in collaboration with teachers, administrators, and mathematics experts. The expectation is that teachers will now incorporate Common Core measures into their instructional practice, which will ultimately prepare students for 21st century demands. Students must be able to think critically, work collaboratively, communicate effectively, think creatively, and make use of technological tools. How do teachers construct environments for students to acquire 21st century skills? This book is designed to help teachers and administrators answer this question. This book includes resources that enable teachers to increase their knowledge of the Common Core State Standards, which is key to students increasing their understanding of mathematics. Not only will teachers have the opportunity to increase their understanding of math content, but they will also learn how to develop standards-based lessons and measure students' understanding of the content using authentic assessment practices. The combination of all of these skills is often known as pedagogical content knowledge. Specialized content pedagogical knowledge is seen as a requisite for effective mathematics instruction (Van Zoest, Stockero, and Taylor 2011). This book is intended as a guide for effective Common Core mathematics instruction.

Parallel Sets of Standards

The CCSS-M represent the intersection of two parallel sets of standards. The content standards define the skills and knowledge in the discipline of mathematics. These standards were constructed on a series of progressions defined by the natural structure of mathematical concepts. Knowledge of students' developmental readiness was also taken into account. The content standards are delineated by grade level for grades kindergarten through eight. Conceptual categories are used at the high school level. The other set of standards that run parallel to the content standards are the standards for mathematical practice. The practice standards are actions, dispositions, and characteristics demonstrated by mathematicians as they negotiate the mathematics content. The practice standards are not a superordinate or subordinate set of standards. The two sets of standards are designed to work together. They should be taught concurrently.

Instructional Shifts

The CCSS-M define a set of mathematical expectations, but they do not dictate a prescribed curriculum. Comprehensive curricula include academic standards, targeted learning objectives, purposeful learning experiences, and the assessment of these experiences. In order to preserve the integrity of the standards during instructional delivery, alignment between the standards and curriculum should incorporate the three instructional shifts of focus, coherence, and rigor.

Focus describes the content that should be emphasized at each grade level. Certain content, commonly called "high-leverage content," has lasting implications for student readiness in mathematics. A majority of instructional time should be allocated to content denoted as "major clusters." The remaining time should then be used to teach supporting and additional clusters. As a result, students will have time to develop an understanding of integral mathematics topics at their unique grade level.

Coherence refers to the relationships among mathematical concepts. It highlights the importance of making connections between and across grade-level standards. All concepts presented should be situated in the larger context of math in order to construct schema and increase retention of information. Creating opportunities for students to see mathematics as a coherent subject eliminates the need for rote memorization of content.

Rigor describes the union of conceptual knowledge, procedural skills and fluency, and the application of knowledge in a real-world context. All three subsets should be represented with equal intensity to create a rigorous exploration of a mathematical concept. In order for students to understand mathematics and meet Common Core expectations, they must have an opportunity to be exposed to many different question types that target each component of rigor. Rigor is not reserved for students at a higher performance level.

The Common Core State Standards in Mathematics, along with the associated instructional shifts of focus, coherence, and rigor, are designed to develop mathematically proficient students. An understanding of what it means to be mathematically proficient may challenge the existing paradigm held by educators.

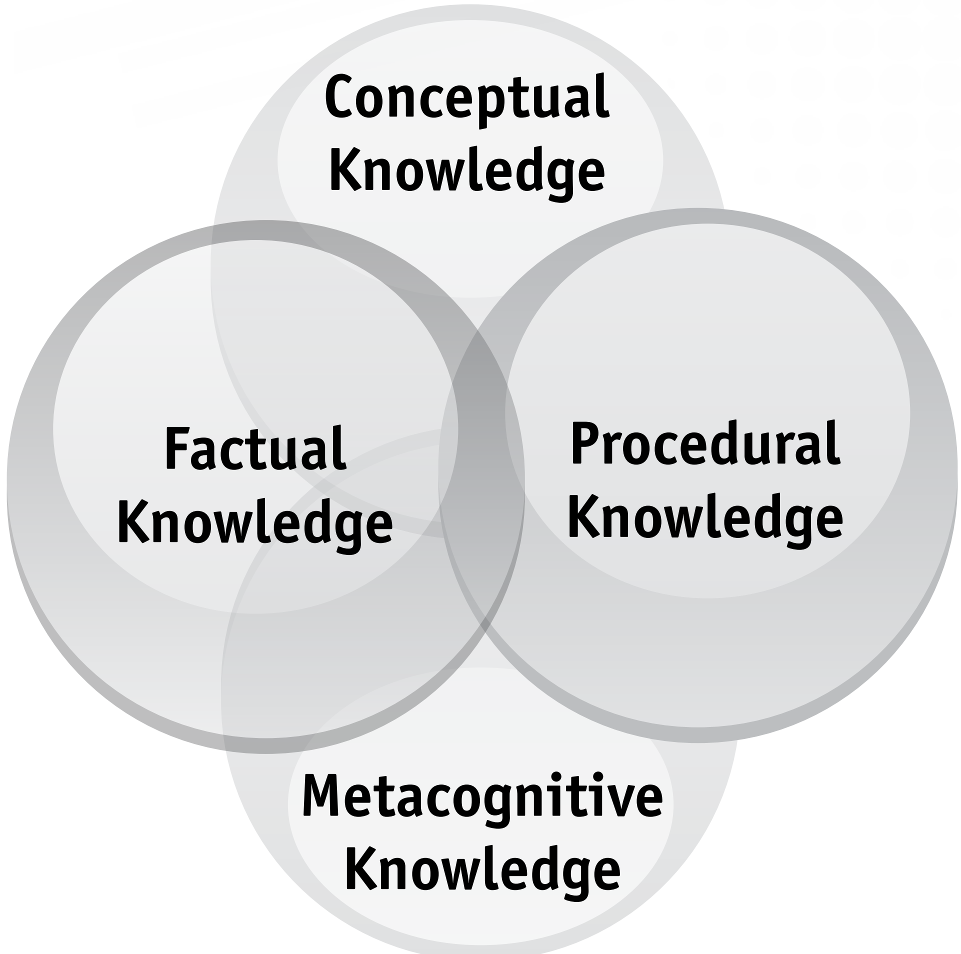
Assessment

The federal government established a common goal of proficiency in mathematics for all students through the policy of No Child Left Behind (US Department of Education 2001). This measure of mathematical proficiency has changed with the creation of the Common Core State Standards (CCSS). A new law, Every Student Succeeds Act (ESSA), signed by President Obama on December 10, 2015, focuses on preparing all students for success in college and careers.

In the past it has been noted by researchers such as Barbeau (1989) that district leaders often viewed mathematics as an established body of rules and procedures, and viewed "doing mathematics" as the mere manipulation of numbers. Previous state assessments reflected this underlying belief. The current assessments created by the Partnership for Assessment of Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortium (SBAC) reflect a more comprehensive view of "doing mathematics." The Common Core has established a new standard of proficiency. This standard includes a student's command of knowledge across multiple knowledge dimensions, in addition to procedural and factual knowledge, which was once the primary focus of mathematics education. Students must now possess conceptual knowledge and metacognitive knowledge.

Under the CCSS, mathematical proficiency is dependent upon the development of all four knowledge dimensions.

The illustration below shows the four knowledge dimensions. Different problem types generally target a specific knowledge dimension, but there is often overlap. Understanding discrete facts may require grouping into overarching ideas known as concepts. Application of concepts requires procedural skills. Reflection upon application and use of concepts incorporates metacognitive skills.



Each dimension of knowledge is defined and described in the chart on page 12. Different question types that target the development of each dimension are also included. The Common Core assessments created by PARCC and SBAC include multiple question types, such as selected response questions (SR) where students answer multiple choice questions, constructed response questions (CR) where students develop their own answer, extended response questions (ER) where students develop their answer and explain, technology enhanced questions (TE) where students use technological tools to answer questions, and performance tasks (PT) where students demonstrate conceptual and procedural knowledge in a real-world setting. A combination of these question types assesses a student's comprehensive understanding of mathematics knowledge across the four dimensions.

Dimension	Description	Question Example
Factual Knowledge	The basic elements that students must know to be acquainted with a discipline or solve problems in it • Knowledge of terminology • Knowledge of specific details and elements	What does "sum" mean? What is the value of the 8 in the number 83?
Conceptual Knowledge	The interrelationships among the basic elements within a larger structure that enable them to function together • Knowledge of classifications and categories • Knowledge of principles and generalizations • Knowledge of theories, models, and structures	What is the relationship between multiplication and division? How does the placement of a digit determine its value?
Procedural Knowledge	How to do something; methods of inquiry and criteria for using skills, algorithms, techniques, and methods • Knowledge of subject-specific skills and algorithms • Knowledge of subject-specific techniques and methods	What is the solution to this problem? $\frac{1}{2} + \frac{3}{4} = \square$ Find the product. $2 \times 3 \times 5 = \square$
Metacognitive Knowledge	Knowledge of cognition in general, as well as awareness and knowledge of one's own cognition • Strategic knowledge • Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge • Self-regulating knowledge	Consider the following problem: Three friends have 18 jelly beans all together. They want to share the jelly beans equally. How many jelly beans should each friend get? What are multiple strategies that can be used to determine the solution? How is this problem familiar to something you've encountered? How did you determine what to do?

How to Use This Book

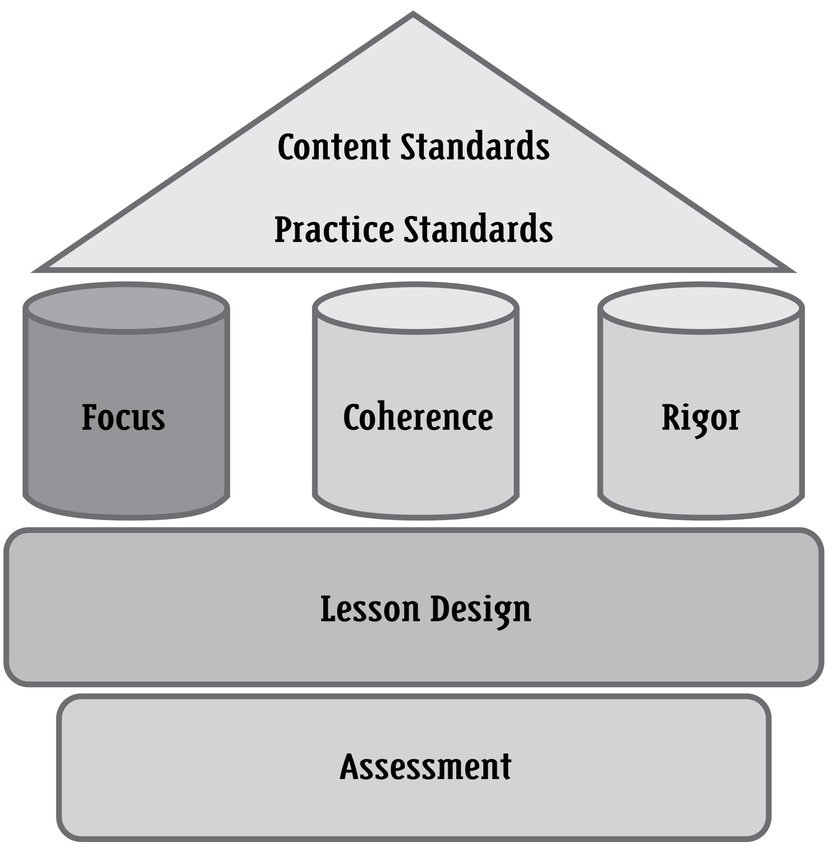
The purpose of this book is to provide practical tools for teachers and administrators to use in creating environments that lead to mathematically proficient students—students who will be able to demonstrate a command of all four knowledge dimensions.

Mathematical rigor in the classroom *for students* includes lessons that target conceptual knowledge, procedural knowledge, and the application of this knowledge in context. Professional development *for teachers* must also include opportunities for teachers to develop all three dimensions of rigor as it applies to the Common Core. This book primarily focuses on teachers' procedural knowledge of standards implementation as they apply the information and resources presented in this book. Procedural questions are targeted, such as:

- *How do I teach the Common Core State Standards in mathematics?*
- *How do I apply the information in this book to my own classroom practice?*

This book provides a series of lessons, templates, and exemplars for practical classroom application. It is essential to note that a teacher's conceptual understanding of the standards is necessary for educators to extend and innovate beyond the ideas in this book. Opportunities should also be provided for educators to construct their understanding of the Common Core Standards and lesson design through their own inquiry and investigation.

Teachers must understand the content standards and the mathematical practice standards in order to develop meaningful mathematics lessons. The units of study created should include the instructional shifts of focus, coherence, and rigor. These shifts will be explained throughout the book. Once lessons are designed that incorporate the Common Core elements, students must demonstrate their understanding through authentic assessment practices. The following illustrative framework represents the relationship between the standards, the Common Core shifts, lesson design, and assessment. The structure of this book and the connection between the chapters is designed based on this relationship.



Overview of This Book

The following is an overview of the chapters in this book. A brief description captures the major ideas and resources in each chapter.

Chapter 1, **Understanding the Standards**, gives a structural overview of the content standards. In order to teach the standards, teachers must know the standards. Tools are provided to deconstruct and analyze the standards to develop a teacher's authentic understanding of the mathematic content.

This chapter also includes planning tools for incorporating the mathematical practice standards into curriculum design and instructional delivery. The alignment between the content standards and the math practice standards is explained. Questioning strategies that target the mathematics practice standards and illustrative prompts are also included.

Exploratory Questions:

1. *How do I increase my understanding of the standards?*
2. *What is the relationship between domains, clusters, and individual standards?*
3. *How do I determine which math practices to target in a lesson?*
4. *How do I teach students to understand the math practices alongside the math content?*

Chapter 2, **Instructional Shift of Focus**, highlights the grade-level foci and grade-level expected fluencies. Major and supporting clusters are defined and explained.

Exploratory Questions:

1. *How do I determine what content should be emphasized at my grade level?*
2. *How do I determine how much time should be allocated to content during curricular planning?*

In **Chapter 3**, **Instructional Shift of Coherence**, it shows how mathematics is a discipline in which all topics are interconnected. Students must understand these connections in order to learn and to not rely upon memorization. Resources are provided to help students make these vital connections.

Exploratory Questions:

1. *How do I use the math clusters to help students understand connections between lessons?*
2. *How might I create opportunities for students to understand how the supporting and/or the additional clusters connect to the major clusters?*

Chapter 4, Instructional Shift of Rigor, gives practical examples of problems that incorporate the three dimensions of rigor. Ideas for striking a balance among the dimensions of rigor are provided. Tools for monitoring instructional rigor are included.

Exploratory Questions:

1. *What are the qualities of a rigorous math lesson?*
2. *How do I determine what math problems to use in order to develop a particular dimension of rigor?*

In **Chapter 5**, **Lesson Design**, a variety of instructional models are provided that show how teachers can design standards-aligned lessons. An auditing tool to help teachers determine the Common Core readiness of their existing lessons is included.

Exploratory Questions:

1. *What are the features of a Common Core lesson?*
2. *How do I develop a unit that balances group investigation with direct instruction?*

Chapter 6, **Assessment**, explains the significant shift in how students must demonstrate their understanding of mathematics. This chapter provides rich examples of assessment question types.

Exploratory Questions:

1. *How will students be assessed using standardized testing measures?*
2. *How might I incorporate various question types in my assessment measures to provide opportunities for students to demonstrate their understanding of math in a meaningful way?*

This book is designed to help you develop a greater understanding of the Common Core State Standards and presents ways to incorporate this knowledge into your classroom practice. As you read this book, consider the relatedness between the different chapters. The questions presented above, in addition to many more, will be explored.

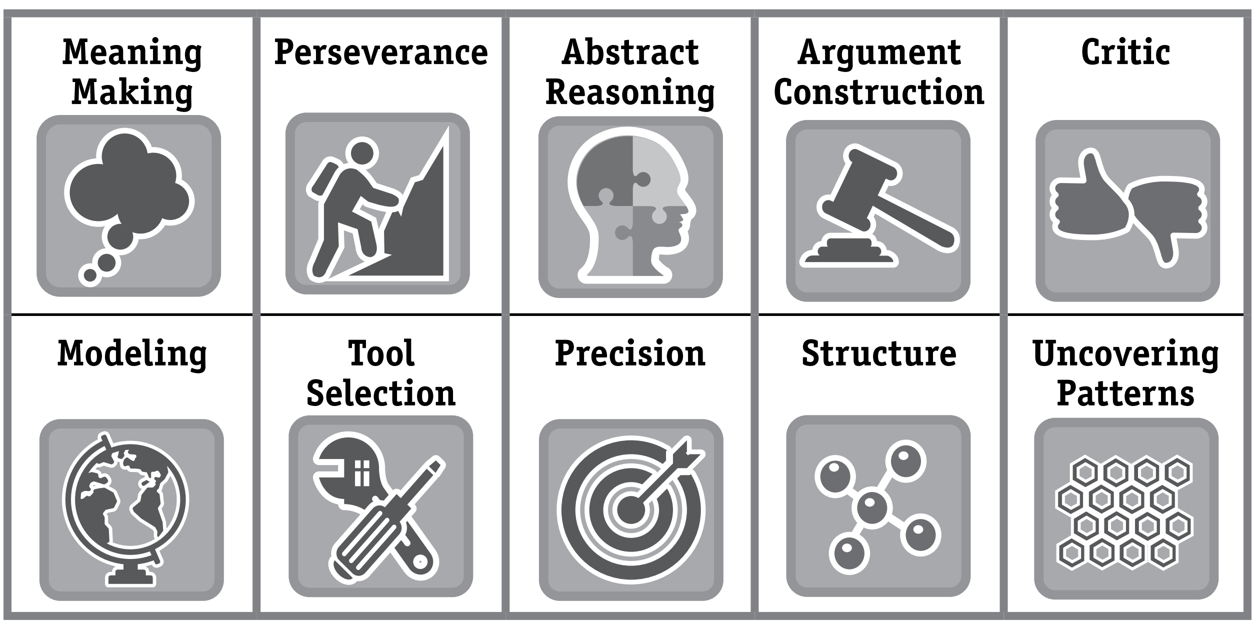
Using Prompts and Questions to Support Student Learning

The use of visual prompts may help students become intentionally aware of the standards for mathematical practice. It is important to discuss the purpose of using prompts with the class. Prompts are used to alert students to a particular practice. Teachers can prominently display these prompts and provide a general overview of each mathematical practice and the corresponding prompt. As teachers and students negotiate math problems, specific prompts can be highlighted by posting the prompt next to the problem. Posting these prompts and discussing how they are used makes the tacit curriculum explicit. After repeated exposure, students become more aware of the mathematical practices and eventually use them in a self-regulated manner. This includes self-selection of particular prompts based on a given math context and the student's justification for their selection. Figure 1.11 represents a series of prompts.

There are times when the math practices may be superordinate in the context of a lesson. For example, if you notice that students are able to develop a mathematical model that represents a given concept but are unable to refine this model through repeated testing, you may want to highlight the process of modeling as the major concept in the lesson. The students would derive an answer but this would be subordinate to a discussion regarding the concept of modeling. This discussion would be used to understand a student's ability to develop conjectures, test these conjectures, and determine the validity of their answers (MP4). Making the mathematical practice central in a lesson should be done periodically to emphasize the importance of the mathematical practices.

Enlarged versions of these visual prompts in color are included on the Digital Resource CD.

Figure 1.11 Visual Prompts



Once the mathematical practices have been selected for a particular task and prompts are visually displayed, questioning strategies by both teachers and students can be used to develop the mathematical practice. Encourage students to generate questions beyond those provided in Figure 1.12.

Figure 1.12 Questions and Student Actions for the Mathematical Practice (MP) Standards

Student Actions	Teacher-Directed Questions
MP1: Make sense of problems and persevere in solving them.	
• Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor student progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask, "Does this make sense?" • Understand various approaches to solutions.	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about ________? • What information is given in the problem? • What is unknown in the problem? • How might you describe the relationship between the quantities? • Describe what you have already tried. What might you change? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize/represent/show ________?
MP2: Reason abstractly and quantitatively.	
• Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and remain flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attend to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is ________ related to ________? • What is the relationship between ________ and ________? • What does ________ mean to you? (*symbol, quantity, diagram*) • What properties might we use to find a solution? • How can this be represented symbolically, pictorially, or in words? • How can you express this answer in the context of the problem?

Student Actions	Teacher-Directed Questions
MP3: Construct viable arguments and critique the reasoning of others.	
• Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? • How can we be sure that ________? • How could you prove that ________? • Will it still work if ________? • What were you considering when ________? • How did you decide to try that strategy? • How did you test whether your approach worked? • What assumptions were made in the argument? • How did you decide what the problem was asking you to find? • Did you try a method that did not work? • What is ambiguous in the argument presented? • What is the same and what is different about ________? • How could you demonstrate a counterexample?
MP4: Model with mathematics.	
• Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply known mathematics to solve everyday problems. • Simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation with either an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask, "How can I represent this mathematically?"	• What questions can I ask of this real-world scenario? • What equation could represent the problem? • Does this model accurately represent this real-world situation? • Have I validated the model through multiple tries? • How will I communicate my findings to the public? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram/number line/chart/table? • Where do you see one of the quantities from the task in your equation or expression? • How would it help to create a diagram/graph/table? • What are some ways to visually represent ________? • What formula might apply in this situation?

Student Actions	Teacher-Directed Questions
MP5: Use appropriate tools strategically.	
• Use available tools to recognize the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen mathematical understanding.	• What mathematical tools can be used to visualize and represent the situation? • What resources are more efficient to help solve the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use a graph/number line/ruler/diagram/calculator/manipulative? • Why was it helpful to use ________? • What can using a ________ show us that using a ________ may not?
MP6: Attend to precision.	
• Communicate precisely and use clear mathematical language when discussing reasoning. • Understand the meanings of symbols used in mathematics and label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem's context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Have you answered all of the parts of the question? • Have you labeled the appropriate parts (axes, units)? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, and properties can you use to explain ________? • How could you check your solution to see if it answers the question?

Student Actions	Teacher-Directed Questions
MP7: Look for and make use of structure.	
• Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complex things as single objects or as being composed of several objects.	• What observations do you make about ________? • What do you notice when ________? • What parts of the problem might you eliminate/simplify? • What patterns do you find in ________? • How do you know if something is a pattern? • What previously learned ideas were useful in solving this problem? • How do the individual parts help you understand the entire problem? • How does this relate to ________? • In what ways does this problem connect to other mathematical concepts?
MP8: Look for and express regularity in repeated reasoning.	
• See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• How does this strategy work in other situations? • Is this always true, sometimes true, or never true? • How would you prove that ________? • What do you notice about ________? • What is happening in this situation? • What would happen if ________? • Is there a mathematical rule for ________? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Once the prompts have been selected based on an understanding of the targeted content, the teacher places the prompts prominently on the board. Associated questions drawn from the chart above are identified. As the teacher asks each question, each prompt is raised to create an association between the prompt and the desired student action.

The following two examples illustrate how the visual prompts and questions can be used in the classroom.

Lesson Sample: *Full of Pie*

Grades 3–5

Cluster(s):

Develop understanding of fractions as numbers.

Emphasis: Major
Supporting
Additional

Standards:

- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (3.NF.3)
- Make sense of problems and persevere in solving them. (MP1)
- Construct viable arguments and critique the reasoning of others. (MP3)
- Look for and make use of structure. (MP7)

Learning Objective:

Students will be able to compare fractions to determine equivalence.

Present the Problem/Cluster

1. Present the following mathematical scenario and cluster for students to investigate. Say, "We will investigate a mathematical scenario to further shape our understanding of fractions as numbers. I want us to think about the following question: When are fractions equivalent?"
2. Allow students time to think independently, then say, "Today, we will explore the concept of equivalent fractions. We are also going to continue to develop our skills as mathematicians—to notice the structure of fractions and to construct viable arguments to defend our solutions. Group A has three pies that will be shared evenly among four students. Group B has two pies that will be shared evenly among three different students. If you want to have the most pie, which group would you join? Present your argument and explain your thinking."
3. Students will then ask clarifying questions. Some examples may be:
    - How does the structure of a fraction help me understand how to approach the problem?
    - How are fractions equivalent if they do not look alike?

Solicit Questions

4. Ask students to generate questions for exploration based on the task. Record questions that students generate. Some questions may be:
    - Are the pies the same size?
    - How can I compare three pieces to four pieces mathematically? I can see that the slice of pie in Group B is larger.
    - Does the pie have to be cut into only thirds or fourths?
5. If needed or desired, ask the following strategic questions to prompt exploration:
    - How did you select a strategy to try? (MP3)
    - What evidence is used to support your position? (MP3)
    - What patterns do you notice? (MP7)

Lesson Sample: *Full of Pie* (cont.)

Grades 3–5

research

Mathematical Research

6. Have students work independently, then in groups to develop strategies/methods to solve the problem and answer the questions that have been generated. A second option involves students immediately working in groups. Examples of strategies students may use include:
   - Students partition pies in multiple ways that may allow them to compare both pies.
   - Students partition pies into thirds and fourths and compare sizes.
7. Circulate around the room while students are working and ask questions targeting the math practices, coherence (connection to cluster and other content), and rigor (concept—why, purpose; procedure—selected algorithm). Note selected solution pathways that will be shared. Suggested questions:
   - What is the relationship between the number of pies and the fraction produced? (Analytical Questions)
   - How does your understanding of fractions help find a solution? (Coherence)
   - What mathematical operations are you using to make sense of the problem? (Rigor)
   - What parts of the problem are you using to compare one group with the next? (MP7)

Share and Summarize

8. Allow students to present their findings from research and their argument. Consider selecting students who illustrate specific solution pathways and solutions that are connected to the solicited questions from earlier in the lesson. For example:
   - I would join Group B because it appears that my slices of pie are larger than those from Group A. Also there are fewer people sharing the pies in Group B, so I will have more of the pie. They are not cut into the same number of pieces so I have to estimate.
9. Allow students to act as respectful critics of each other's work by asking the following questions at various points throughout the presentations.
   - You are assuming that you have more based on the size of the pieces. How do you prove this with evidence?
   - I believe that each person in Group A has more pie because I have sliced each pie using the same number of slices to compare.
10. Select solution pathways and connect to solicited questions in multiple ways, including:
    - How are the solution pathways similar/different?
    - How does this solution emphasize the concept of fraction equivalence?
    - What questions from the board have we answered?

Recycle and Forecast

11. Highlight the studied cluster from the lesson and allow students to discuss the meaning that was created. Students can generate additional questions as a result of the research. Examples include:
    - It is not always easy to cut the pie or any shape into the same number of pieces in order to compare. How might we compare fractions with different denominators using some type of rule?
    - How are common unit fractions helpful in comparing problems?
12. Ask students new questions as a result of the exploration and provide questions that will be explored in future lessons. Some examples include:
    - How might you compare different parts or fractions that refer to the same whole?
13. Close the lesson with a journal prompt. Say, "How does this lesson increase your understanding of equivalent fractions? Record your response in your journal."

NOTES

You Try It!

1. Download the Direct Instruction Lesson template from the Digital Resource CD (filename: directinstruct.pdf).
2. Review the direct instruction model and sample lessons (pages 156–165).
3. Use the Direct Instruction Lesson Template to create your own lesson.
4. After teaching your lesson, reflect on ways you can improve upon planning or teaching the lesson to help students meet the learning objective.

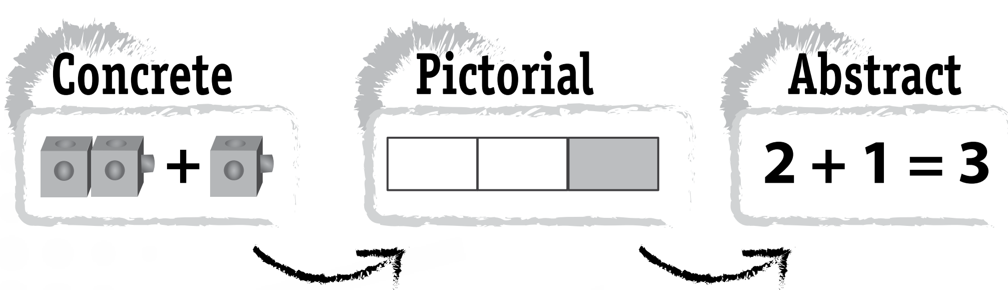
Common Core Instructional Strategies

Within any given unit of study or a specific lesson, instructional strategies are embedded to increase a student's comprehension of the math topic under investigation. Several strategies are referenced in the Common Core State Standards (2010) ranging from the use of concrete models to pictorial drawings to decontextualized abstract representations. The combination of contextualized and decontextualized strategies increases a student's conceptual understanding of mathematics, resulting in the ultimate goal—application of mathematics in the real world.

Standard for Mathematical Practice 2—Reason abstractly and quantitatively—emphasizes the importance of a student's ability to apply abstract mathematical representations and symbols from a concrete situation and the converse, to be able to describe a concrete context given an abstract representation. It is known that students' knowledge is strongest when they can connect real-world situations, manipulatives, pictures, and spoken and written symbols (Lesh 1990).

Meaningful learning occurs when students are able to organize new concepts in their long-term memory by making connections between familiar concepts. Concrete, everyday representations often serve as these familiar concepts and act as anchors for new concepts.

Curricular resources will often suggest prescribed steps when exploring the relationship between the concrete, pictorial representation, and abstract ideas.



However, it is important to note that students should have the opportunity to not only explore concrete to abstract pathways but also abstract to concrete pathways.

Scientific-based research has not shown what modes of presentations are crucial and what sequence of representations should be used before abstract symbols are introduced (Baroody 1989; Clements 1999). Teachers should be careful about blindly adhering to a concrete-pictorial-abstract sequence, especially when more than one way of thinking about a concrete context exists.

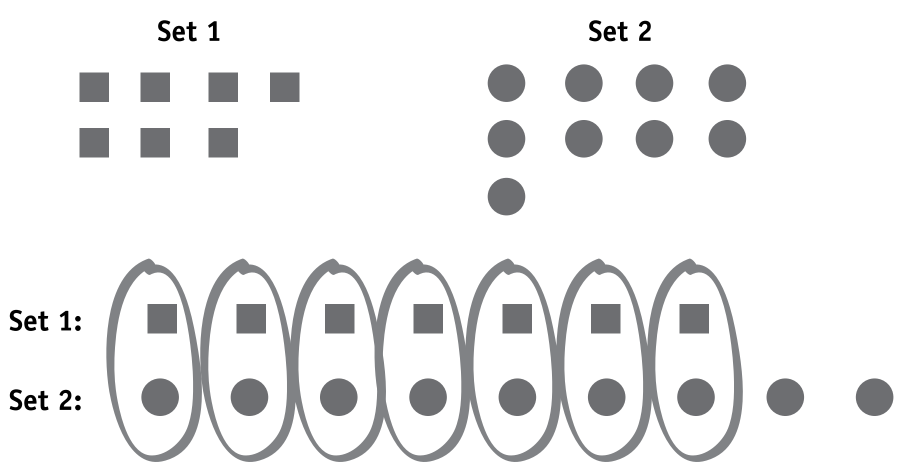
Mathematical Models vs. The Modeling Process

There is a distinctive difference between the use of concrete models to understand mathematical concepts and the process of mathematical modeling. A mathematical model is an isolated representation of a concept. It may be a physical construct or mathematical symbols that represent the concept. A mathematical model is a subset of the process of mathematical modeling.

Mathematical modeling is a cyclical process in which students develop ideas and conjectures about a particular real-world mathematics scenario. They then create a mathematical model that best represents the concept. This model is then tested and refined in an iterative fashion. The results are then explained and justified in the context of the mathematics scenario. The mathematical model is only a component of the entire process of modeling.

Common Core explicitly mentions different instructional strategies, some of which are concrete models that may be applied relating to unique standards. The figures that follow highlight several of these strategies, many of which are instructional models. Though these instructional strategies are listed for a particular grade, they are advantageous to use across grade levels based on students' readiness level and responsiveness to the strategy being implemented.

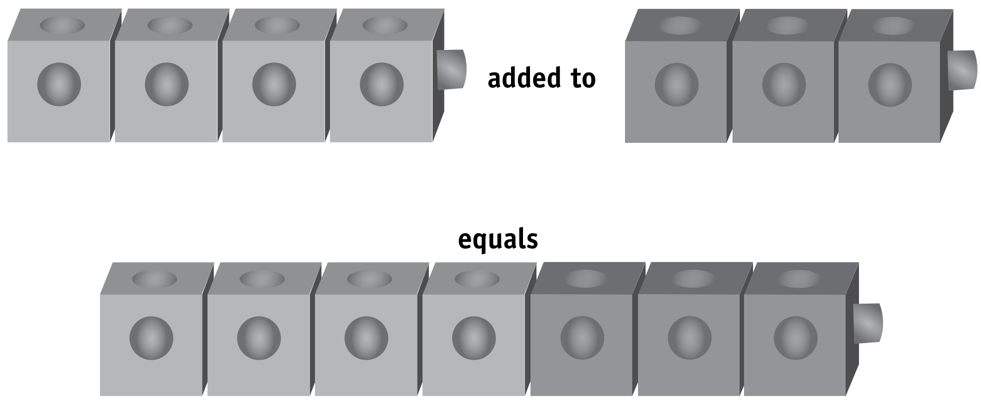
Figure 5.6 Mathematical Strategies K–8

<table>
<tr><th colspan="2">Kindergarten</th></tr>
<tr><td>Standard</td><td>K.CC.6</td></tr>
<tr><td>Instructional Strategy</td><td>Matching and Counting</td></tr>
<tr><td colspan="2">One-to-one correspondence is used to pair elements from each set. Students will determine which elements are not matched and therefore can identify which set has more or less elements. Students may also count each set and compare the values to determine which set has more or less.


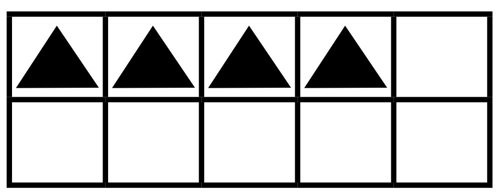
Set 2 has two elements that are not matched. Therefore, set 2 has more elements.</td></tr>
</table>

Standard(s)	K.OA.1, K.OA.2, K.OA.3, K.OA.4, K.NBT.1
Instructional Strategy	Use of objects, fingers, mental images, drawings, and sounds (e.g., claps). Use of a ten frame or a number bond.

Students use objects and drawings to determine addition and subtraction problems that involve putting together, adding to, taking apart, and taking from.

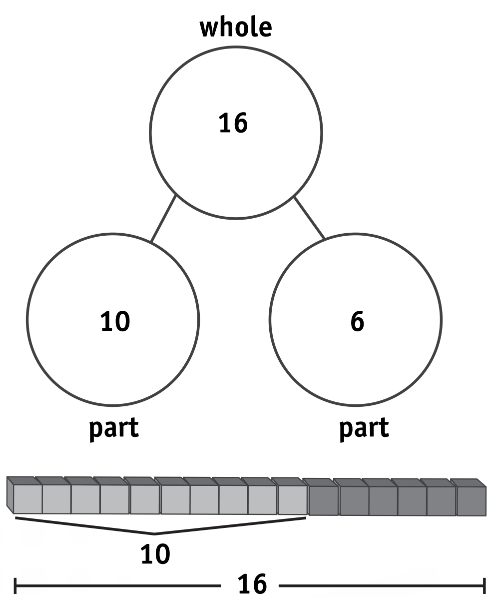


Students place objects on a ten frame (physical one or drawing) and determine how many more are needed to compose ten. Taking away elements from a full ten frame can be used to show how 10 is decomposed into two numbers.



Six additional triangles are needed to make 10.

Number bonds can be used to show the relationship between addition and subtraction and the relatedness between a whole number and its parts.



10 + 6 = 16
6 + 10 = 16
16 − 10 = 6
16 − 6 = 10