

Focused Mathematics Intervention

Lessons and Activities

Level 3

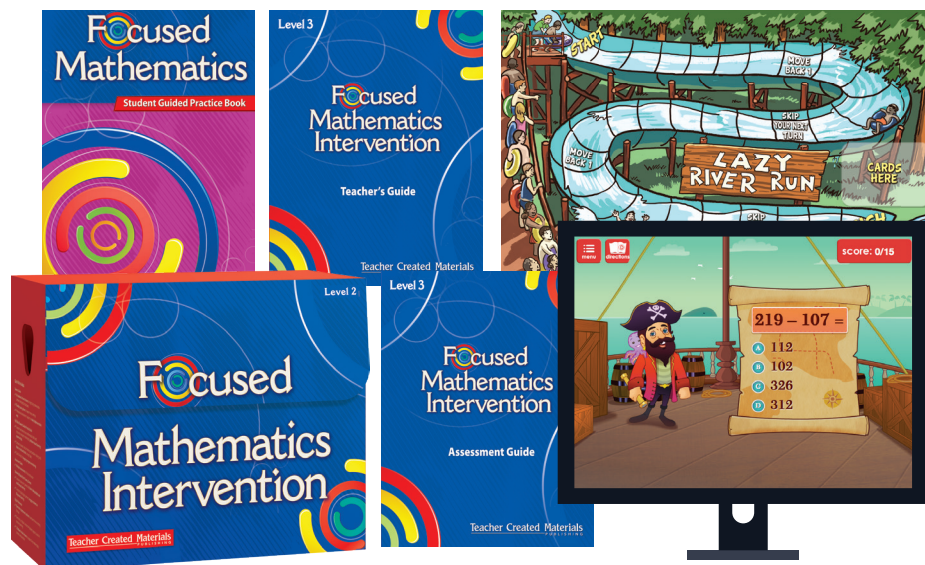
Table of Contents

Teacher's Guide Cover (1 page)

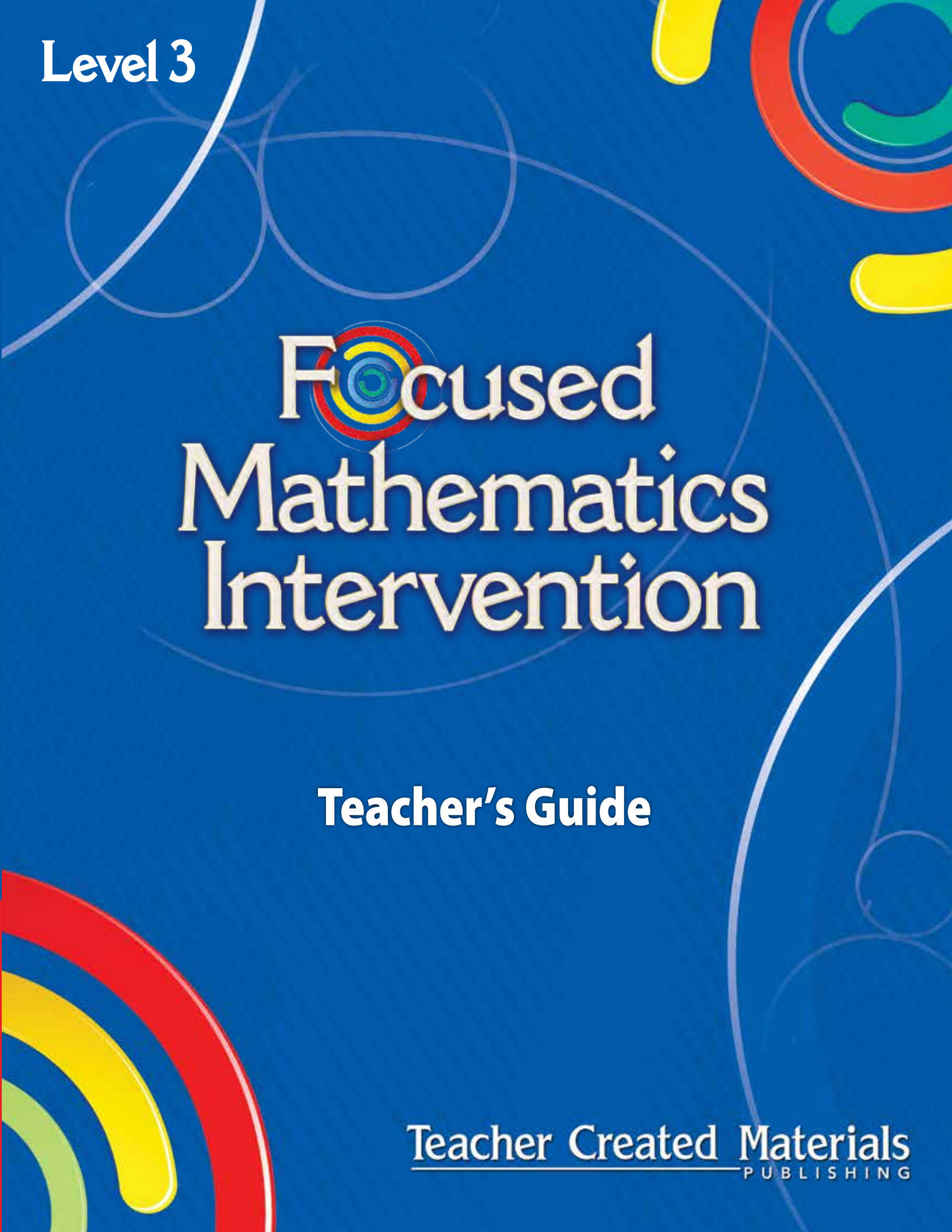
Teacher's Guide Table of Contents (1 page)

How to Use This Product (3 pages)

Lesson Plan (17 pages)



Level 3



Focused Mathematics Intervention

Teacher's Guide

Teacher Created Materials
PUBLISHING

Table of Contents

Welcome

Program Welcome.....	4
----------------------	---

Research

Research on Mathematics Intervention	6
The Need for Intervention.....	7
Response to Intervention in Mathematics....	8
Components of Effective Mathematics Interventions	10
High-Yield Strategies for Increasing Student Achievement	12
Using Technology to Improve Mathematical Learning.....	13
Using Games to Motivate Struggling Math Learners	14
Assessment	15

Best Practices

Components of Effective Mathematics Intervention Programs.....	17
Differentiation	19
Differentiating by Specific Needs	19
Developing Academic Vocabulary.....	21
Academic Vocabulary	21
Developing Math Skills Using Concrete Models	22
Developing Mathematical Problem-Solving Skills.....	24
Why We Teach Problem Solving.....	24
Making Connections	24
A Problem-Solving Framework.....	25
Math in the Real World	28
Developing Math Fluency Skills	29

How to Use This Product

Kit Components.....	31
Getting Started.....	32
Teaching a Lesson	33
Using the Math Fluency Games	35
How to Organize and Manage Games.....	35
Playing the Math Fluency Game Sets.....	36
Playing the Digital Math Fluency Games.....	39
Using the Technology Options.....	40

Planning for Intervention

Pacing Plans	41
Correlations	45
Introduction to Correlations	45
Standards Correlations	46
Series Scope and Sequence	49

Lessons

Lesson 1: Addition Strategies.....	61
Lesson 2: Addition Patterns	69
Lesson 3: Subtraction Strategies	77
Lesson 4: Elapsed Time	85
Lesson 5: Multiplication Models	93
Lesson 6: Partitive Division	101
Lesson 7: Measurement Division.....	109
Lesson 8: Problem Solving with Multiplication.....	117
Lesson 9: Multiplication Patterns	125
Lesson 10: Problem Solving with Division ..	133
Lesson 11: Using Inverse Relationships ...	141
Lesson 12: Multiplying with the Associative Property	149
Lesson 13: Two-Step Word Problems	157
Lesson 14: Square Units.....	165
Lesson 15: Measuring with Square Units ..	173
Lesson 16: Finding the Area Formula.....	181
Lesson 17: Problem Solving with Area	189
Lesson 18: Area of Rectilinear Figures	197
Lesson 19: Recognizing Perimeter	205
Lesson 20: Unit Fractions	213
Lesson 21: Building Fractions from Unit Fractions.....	221
Lesson 22: Plotting Fractions on Number Lines.....	229
Lesson 23: Equivalent Fractions	237
Lesson 24: Fractions Equal to One	245
Lesson 25: Comparing Fractions 1	253
Lesson 26: Comparing Fractions 2	261
Lesson 27: Scaled Graphs.....	269
Lesson 28: Volume and Mass	277
Lesson 29: Line Plots	285
Lesson 30: Quadrilaterals.....	293

Appendices

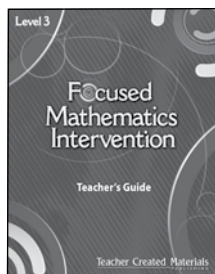
Appendix A: References Cited	301
Appendix B: Teacher Glossary.....	304
Appendix C: Digital Resources Charts	313

Kit Components

HOW TO USE
THIS PRODUCT

Teacher's Guide

30 easy-to-use, standards-based lesson plans



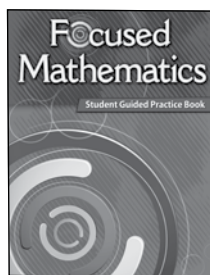
3 Digital Math Fluency Games

Focus on mathematical skills and strategies, and are on the Digital Resources USB Device



Student Guided Practice Book

Full-color student activities

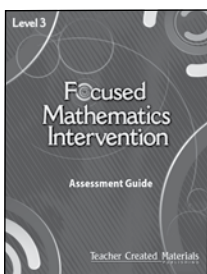


Digital Resources

- PDFs of all student materials, game sets, activity sheets, assessments, etc.
- PDFs of teacher resources
- Digital Math Fluency Games
- Electronic versions of the Pretest, Posttest, Performance Tasks, and reporting tools

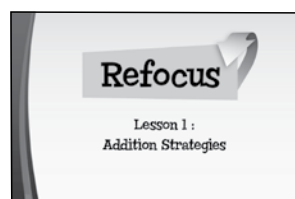
Assessment Guide

Includes a pretest, posttest, performance tasks with assessments, and the answer key for the *Student Guided Practice Book*



Refocus Mini Lesson

Provide as PowerPoint® and PDF files



3 Math Fluency Game Sets

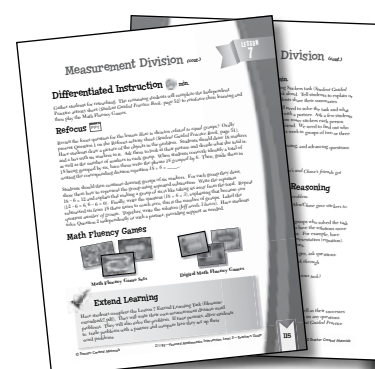
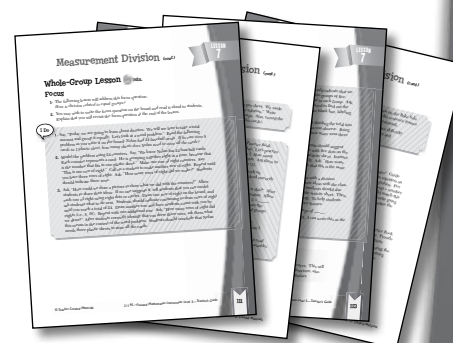
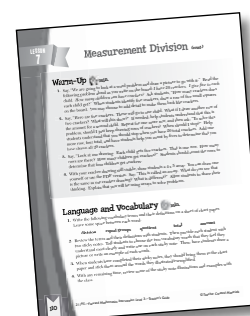
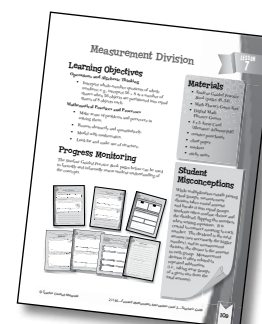
Include a game board, directions, an answer key, and game pieces



Teacher's Guide

Each 8-page lesson is organized in a consistent format for ease of use. Teachers may choose to complete some or all of the lesson activities to best meet the needs of their students. Lesson materials can be utilized flexibly in a variety of settings. For example, modeling with a small group, using printed materials with a document camera, or using PDF materials on a digital platform, such as an interactive whiteboard. Each lesson includes:

- an overview page with key information for planning
- key mathematics content standards covered
- key mathematical practices and processes addressed
- an overview providing teacher background or student misconceptions
- a Warm-Up activity to build students' recall of important mathematical concepts
- a whole-class Language and Vocabulary activity
- time markers to indicate the approximate time for instruction
- a whole-class section focusing on the key concept/skill being taught
- use of the gradual release of responsibility model in the Whole-Group lesson section
- differentiation strategies to support and extend learning with the Refocus lesson and Extend Learning activity
- math fluency games that motivate students to develop and reinforce mastery of basic skills
- a Math in the Real World concept task activity

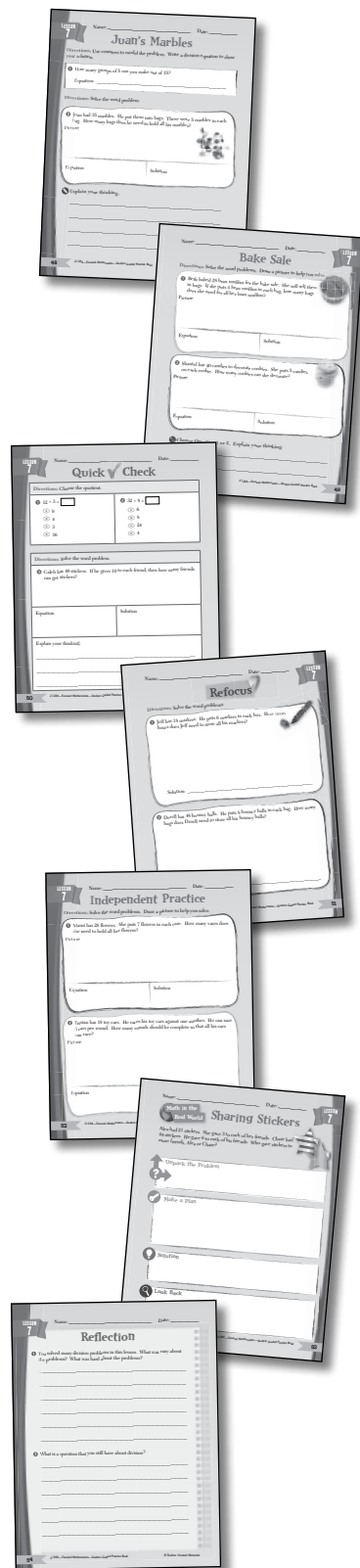


Teaching a Lesson (cont.)

Student Guided Practice Book

Each lesson in the *Teacher's Guide* has seven corresponding student pages in the *Student Guided Practice Book*:

- a We Do activity to support the gradual release of responsibility model
- a You Do activity to facilitate independent practice
- a Quick Check to easily monitor students' progress
- a Refocus activity for students who need more instruction
- an Independent Practice page to reinforce mathematical content taught in the lesson
- a Math in the Real World concept task for students to apply the math concept in a real-life scenario
- a Reflection page for students to share their mathematical understanding



Partitive Division

Learning Objectives

Operations and Algebraic Thinking

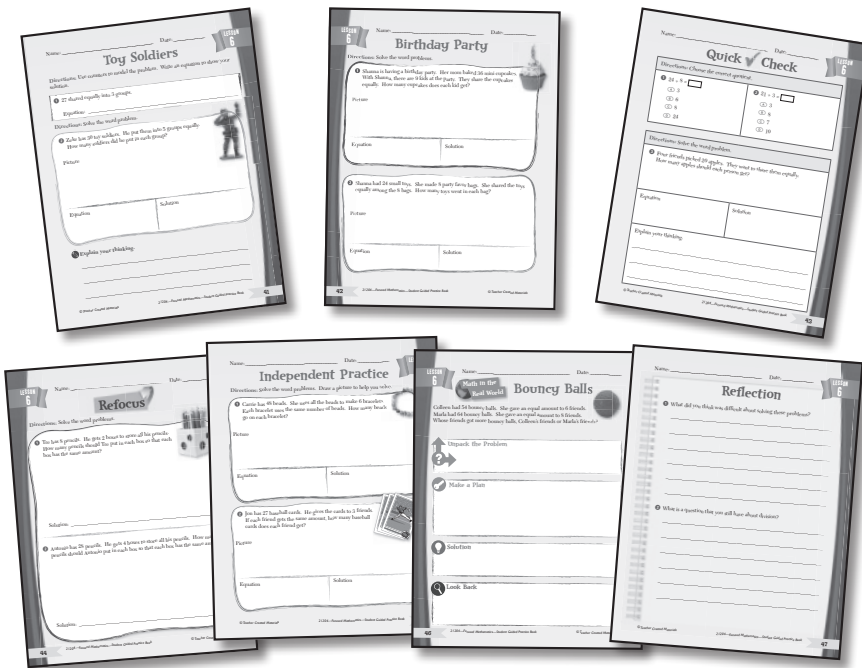
- Interpret whole-number quotients of whole numbers; e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares.

Mathematical Practices and Processes

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

Progress Monitoring

The *Student Guided Practice Book* pages below can be used to formally and informally assess student understanding of the concepts.



Materials

- *Student Guided Practice Book* (pages 41–47)
- Math Fluency Game Sets
- Digital Math Fluency Games
- counter punchouts
- chart paper
- markers
- cups or jars (optional)

Student Misconceptions

While multiplication entails joining equal groups, partitive division (also known as the *fair share model*) takes a total amount and shares the amount fairly or equally. Students often confuse divisor and the dividend, flipping the numbers when writing equations. It is crucial to connect meaning to each number. The dividend is the total amount (not necessarily the bigger number) and, in partitive division, the divisor is the number of groups. In other words, $\text{total amount} \div \text{number of groups} = \text{how many go in each group}$.

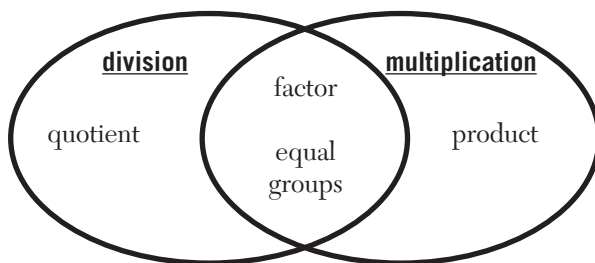
Partitive Division *(cont.)*

Warm-Up 10 min.

1. Have eight students come to the front of the room. Ask, “How could I put these students into two equal groups?” Allow students to share their ideas. Model students’ suggestions, ultimately ending with two groups of four students. Then say, “We put eight students into two groups. How many students are in each group?” Students should indicate that there are four students in each group.
2. Say, “I can use a division equation to show what the students modeled. We start with a number and then share that number equally into a certain number of groups.” Write $8 \div 2 = 4$. Say, “I can read this as 8 divided by 2 is 4, or 8 divided into 2 groups is the same as 4 in each group.”
3. Gather a group of six students and model how to divide them into three equal groups. Use the same process that you followed in Steps 1–2 to represent the modeling with a division equation ($6 \div 3 = 2$).

Language and Vocabulary 10 min.

1. Write the following terms along the top of a sheet of chart paper.
division equal groups quotient multiplication factor product
2. Review the definition of each word with students. Then, draw a large Venn diagram below the list of terms. Say, “We have two big ideas that relate to our vocabulary terms. One is *division*. The other is *multiplication*.” Write these labels at the top of each of the two categories on the Venn diagram and cross them out on the list on the chart paper.
3. Say, “All our vocabulary terms have to do with either division, multiplication, or both. We can add them to our Venn diagram in one of these categories, to show how the vocabulary terms are related.” Say a term aloud, and have students share their ideas about where it should be added to the Venn diagram and why. Your completed Venn diagram should look like the following:



Partitive Division *(cont.)*

Whole-Group Lesson (40) min.

Focus

1. The following lesson will address this focus question:
How is division related to equal groups?
2. You may wish to write the focus question on the board and read it aloud to students. Explain that you will revisit the focus question at the end of the lesson.

I Do

1. Say, "Today, we are going to examine different ways of taking a total amount and sharing it fairly or equally. Let's look at a word problem." Write the following problem on the board as you read it aloud: *Andrea is organizing her room. She decides to put her 24 toys into 6 buckets evenly. How many toys should she put into each bucket?*
2. Say, "Let's unpack this problem. We know that Andrea has 24 toys. She is passing them out to six buckets. Let's use counters as part of our plan to solve." Gather 24 counters and put them into six groups, distributing them one at a time into each group until you run out. As an option, you could also pass out the counters into individual cups or jars, to model the buckets. Say, "When we divided the 24 counters into six groups, we put four counters in each group."
3. Say, "Now, let's make a drawing to show the problem. I will draw circles to show the groups. How many circles should I draw?" Students should indicate six circles. Say, "Now, we can draw a dot in each group until we count to 24." Have students count aloud with you as you add dots into each large circle. After you draw 24 dots, ask students to count the number in each circle. Students should indicate that there are four dots in each circle.

Language Support

As you model, emphasize words such as *evenly* and *equally*. This is a key idea for students to understand in partitive division problems, and students must have a clear understanding of vocabulary related to the idea of creating equal groups.

Partitive Division *(cont.)*

Whole-Group Lesson *(cont.)*

I Do
(cont.)

4. Say, "We started with the total, 24. We divided it into six groups and found that there are four in each group. I can write a division equation to show this." Write $24 \div 6 = 4$. Label each term (24: *total*; 6: *groups*; 4: *amount in each group*).
5. Act out and say, "Let's distribute the toys again. Do you think we can distribute more than one at a time to make it go faster?" Have students share their ideas. Suggest distributing two toys at a time. Draw another six circles, and have a student write a 2 in each circle, counting as you go: two, four, six, etc. Say, "That went a little faster. Each group has two and two, or four. We found, again, that 24 divided into six groups gives four in each group."

We Do

1. Refer students to the Toy Soldiers activity sheet (*Student Guided Practice Book*, page 41). Give students counters or other counting manipulatives for this part of the lesson. Say, "Let's practice modeling and solving division equations together. We will begin with Question 1: *27 shared equally into 3 groups*. How will we model this with our counters?"
2. Allow students to work on modeling the problem with a partner. Observe students to see how they are solving with the counters. Select one pair of students that completed the task correctly to model how to distribute the 27 counters into three groups. Ask, "How many counters did we put in each group?" Students should respond that they put nine counters in each group. Model how to write this as a division equation ($27 \div 3 = 9$), and have students do the same on their activity sheets. Label 27 as the *total*, 3 as *groups*, and 9 as *amount in each group*.
3. Say, "Now, let's examine a word problem together." First, read the problem: *Zeke has 30 toy soldiers. He put them into 5 groups equally. How many soldiers did he put in each group?* Say, "Let's unpack this problem. What type of problem is this? How do you know?" Students should indicate that this is a division problem because it involves taking a total and sharing it into equal groups.

Partitive Division (cont.)

Whole-Group Lesson (cont.)

We Do
(cont.)

4. Say, “Now, we need a plan to solve. Let’s use counters to model this problem. What do we know about the problem?” Students should indicate that Zeke has 30 soldiers, and he puts them into groups equally. Say, “In this problem, Zeke makes groups. How many groups will we make with our counters?” Students should indicate that we should make five groups. Say, “We need to find out how many soldiers go in each group.”
5. Allow students to model distributing the 30 counters into five groups independently as you observe. Bring the class back together and have a student model for the class to see. Ask students how many counters are in each group (6).
6. Say, “Let’s use a drawing to check our solution. How can we show the five groups?” Allow students to share their thinking. They should suggest making five large circles. Ask, “How will we show distributing the 30 soldiers to each group?” Help students recognize that you can draw dots one at a time in the circles until you reach 30. Have students complete this drawing in the *picture* box on the activity sheet. The drawings should look like the following:



7. Ask, “How many dots did you draw in each group?” Students should identify six dots, the same solution modeled with the counters. Say, “We found that when we share 30 into five groups we have six in each group. How can we write this as a division equation? What is the solution to our word problem?” Have students write the equation ($30 \div 5 = 6$) and solution (*Zeke put 6 soldiers in each group*) on the activity sheet. Then, have students explain how they solved. To help students explain their reasoning, provide them with the following sentence frames:
 - *There are _____ soldiers. They were divided into _____ groups.*
 - *My model shows _____ soldiers go in each group. I can write this as the division equation _____.*

Partitive Division *(cont.)*

Whole-Group Lesson *(cont.)*

You Do

1. Tell students they will now work on sharing division on their own on the Birthday Party activity sheet (*Student Guided Practice Book*, page 42). Provide the sentence frames from Step 7 of the We Do section to help students explain their reasoning.
2. Have students share their equations and reasoning. If students have difficulty explaining their reasoning, remind them to use the sentence frames and vocabulary terms.

Closing the Whole-Group Lesson

Revisit the focus question for the lesson: *How is division related to equal groups?* Guide students in discussing how division is connected to equal groups. Write a division equation on the board, for example, $40 \div 5 = 8$. Ask students to work with you to label each term in the equation. For partitive division, this equation means total amount $\div$ number in groups = number in each group. Then, ask students to think of a situation or word problem that would match this equation. For example, I have 40 flowers. I divide them into 5 groups to give away to 5 friends. Each group has 8 flowers. Have several students share scenarios that contextualize the equation.

Progress Monitoring (5) min.

1. Have students complete the Quick Check activity sheet (*Student Guided Practice Book*, page 43) to gauge student progress toward mastery of the Learning Objectives. Provide students with unlined paper to show their work on the selected response questions.
2. Based on the results of the Quick Check activity sheet and your observations during the lesson, identify students who may benefit from additional instruction in the Learning Objectives. These students will be placed into a small group for reteaching. See instructions on the following page.

Partitive Division (cont.)

Differentiated Instruction (20 min.)

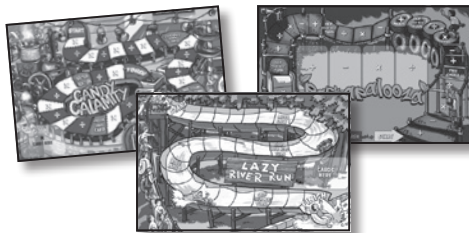
Gather students for reteaching. The remaining students will complete the Independent Practice activity sheet (*Student Guided Practice Book*, page 45) to reinforce their learning and then play the Math Fluency Games.

Refocus

Revisit the focus question for the lesson: *How is division related to equal groups?* Orally present Question 1 on the Refocus activity sheet (*Student Guided Practice Book*, page 44). Have students draw a picture of the objects in the problem. Students should draw eight pencils and two boxes. Ask them to look at their picture and decide what the total is, as well as the number of groups that the total is shared equally into. When students correctly identify a total of eight being shared equally into two, have them write the phrase *8 shared equally into 2 groups*. Ask students to solve by using counters or drawing a picture to represent sharing the eight pencils between the two boxes. Then, guide them in writing the corresponding division equation $8 \div 2 = \underline{\quad}$. Students should then write the quotient (4) and the solution (*Tre should put 4 pencils in each box*).

Have students solve Question 2 independently or with a partner, providing support as needed. Encourage them to use the same steps that you used for Question 1.

Math Fluency Games



Math Fluency Game Sets



Digital Math Fluency Games



Extend Learning

Have students complete the Lesson 6 Extend Learning Task (filename: extendtask6.pdf). They will write their own sharing division word problems. They will also solve the problem. If time permits, allow students to trade problems with a partner and compare how they set up their word problems.

Partitive Division (cont.)

Math in the Real World (30 min.)

1. Refer students to the Math in the Real World: Bouncy Balls task (*Student Guided Practice Book*, page 46). Have a student read the task aloud. Tell students to explain or summarize the task to their partner. Have a few students share their summaries.
2. Ask students to think about what information they will need to solve the task and what the task is asking them to do. Then, have them share with a partner. Ask a few students to share out. Students should identify that we know how many bouncy balls each girl had and the number of friends she shared them with equally. We need to find out whose friends got more. Have students work in groups of two or three to complete the task.
3. As students are working, circulate and ask focusing, assessing, and advancing questions:
 - *How many balls did Colleen share? How many friends did she share with? How many balls did Marla share? How many friends did she share with?*
 - *How can you model this problem with a picture?*
 - *What equations can you use to show how many bouncy balls Colleen and Marla shared with each friend?*

Sentence Frames for Explaining Reasoning

- *Both girls are sharing equally, so this is a _____ problem.*
 - *I used the equation _____ to show how many balls Colleen's and Marla's friends got.*
 - *_____’s friends got more bouncy balls.*
4. Observe how students are solving the task, and choose a few groups who solved the task in different ways to share their solutions and reasoning. Begin with less complex solutions (e.g., distributing the bouncy balls one by one), and then see if any students tried to distribute them by twos. Make sure students explain their reasoning as they share solutions.
 5. As groups are sharing their solution paths, reasoning, and strategies, ask questions:
 - *What was challenging about this problem? How did you solve those challenges?*
 - *How is this strategy similar to one that we have seen in a previous task?*
 - *Which strategy makes the most sense to you? Why?*

Lesson Reflection (5 min.)

Have students write about what they learned about sharing division problems, and provide feedback on any questions they still have about the content on the Reflection activity sheet (*Student Guided Practice Book*, page 47).

Name: _____ Date: _____

Toy Soldiers

Directions: Use counters to model the problem. Write an equation to show your solution.

- 1 27 shared equally into 3 groups.

Equation: _____

Directions: Solve the word problem.

- 2 Zeke has 30 toy soldiers. He put them into 5 groups equally. How many soldiers did he put in each group?

Picture



Equation

Solution

 Explain your thinking.

Birthday Party

Directions: Solve the word problems.



- 1 Shanna is having a birthday party. Her mom baked 36 mini cupcakes. With Shanna, there are 9 kids at the party. They share the cupcakes equally. How many cupcakes does each kid get?

Picture

Equation

Solution

- 2 Shanna had 24 small toys. She made 8 party favor bags. She shared the toys equally among the 8 bags. How many toys went in each bag?

Picture

Equation

Solution

Quick Check

Directions: Choose the correct quotient.

1 $24 \div 8 = \square$

(A) 3

(B) 6

(C) 8

(D) 24

2 $21 \div 3 = \square$

(A) 3

(B) 8

(C) 7

(D) 10

Directions: Solve the word problem.

- 3 Four friends picked 20 apples. They want to share them equally. How many apples should each person get?

Equation

Solution

Explain your thinking.

Refocus

Directions: Solve the word problems.



- 1 Tre has 8 pencils. He gets 2 boxes to store all his pencils. How many pencils should Tre put in each box so that each box has the same amount?

Solution: _____

- 2 Antonio has 28 pencils. He gets 4 boxes to store all his pencils. How many pencils should Antonio put in each box so that each box has the same amount?

Solution: _____

Name: _____

Date: _____

LESSON
6

Independent Practice

Directions: Solve the word problems. Draw a picture to help you solve.

- 1** Carrie has 48 beads. She uses all the beads to make 6 bracelets. Each bracelet uses the same number of beads. How many beads go on each bracelet?



Picture

Equation

Solution

- 2** Jon has 27 baseball cards. He gives the cards to 3 friends. If each friend gets the same amount, how many baseball cards does each friend get?



Picture

Equation

Solution



Bouncy Balls



Colleen had 54 bouncy balls. She gave an equal amount to 6 friends.
Marla had 64 bouncy balls. She gave an equal amount to 8 friends.
Whose friends got more bouncy balls, Colleen's friends or Marla's friends?



Unpack the Problem



Make a Plan



Solution



Look Back and Explain

Name: _____

Date: _____

LESSON
6

Reflection

- 1 What did you think was difficult about solving these problems?

- 2 What is a question that you still have about division?

Pretest

1. Marco has 3 jars. He put 8 marbles in each jar. Which equation shows the total number of marbles that Marco put into the jars?

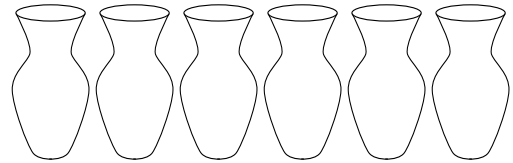
(A) $8 \div 8 = 1$
1 marble

(B) $3 \times 2 = 6$
6 marbles

(C) $3 + 8 = 11$
11 marbles

(D) $3 \times 8 = 24$
24 marbles

3. Terry has 48 flowers. He wants to place them all equally into 6 vases. How many flowers will be in each vase?



(A) 42 flowers

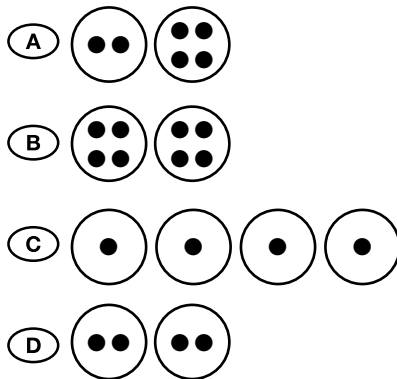
(B) 6 flowers

(C) 54 flowers

(D) 8 flowers

2. Which picture matches this multiplication problem?

$$2 \times 4 = 8$$



4. Ms. Ross has some bowls that she wants to fill with oranges. She has 40 oranges. She puts 5 oranges in each bowl. How many bowls did she fill?

(A) 8 bowls

(B) 25 bowls

(C) 9 bowls

(D) 45 bowls

Name: _____ Date: _____

Performance Task 1: Pancake Breakfast

Part A

There will be a pancake breakfast at Park View School. Eight tables are available. Each table can seat 6 people. How many people can be seated at the breakfast? How do you know this?

Solution: _____

Explain your solution.
